Handbook of Electrochemical Impedance Spectroscopy. ELECTRICAL CIRCUITS CONTAINING CPEs

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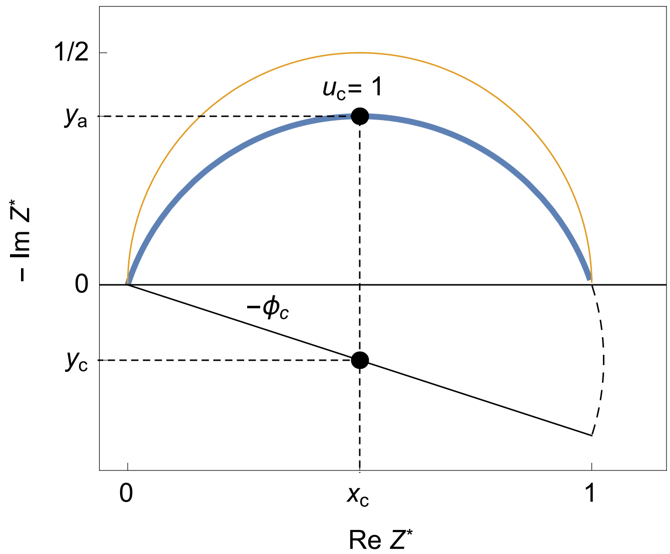
Handbook
of
Electrochemical Impedance Spectroscopy



ELECTRICAL CIRCUITS CONTAINING CPEs

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Contents

Chapter 1

Circuits containing one CPE

1.1 Constant Phase Element (CPE), symbol Q

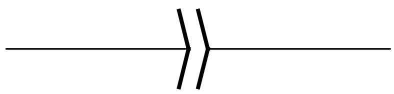

Figure 1.1: Most often used symbol for CPE (see also the Appendix A).

$$Z = \frac{1}{Q\,(\mathrm{i}\,\omega)^{\alpha}}\ ,\ \mathrm{Re}\,Z = \frac{c_\alpha}{Q\,\omega^{\alpha}}\ ,\ \mathrm{Im}\,Z = -\frac{s_\alpha}{Q\,\omega^{\alpha}}$$

$$c_\alpha = \cos(\frac{\pi\,\alpha}{2})\ ,\ s_\alpha = \sin(\frac{\pi\,\alpha}{2})$$

$$|Z| = \frac{1}{Q\,\omega^{\alpha}}\ ,\ \phi_Z = -\frac{\pi\,\alpha}{2}$$

The Q unit ($\mathrm{F\,cm^{-2}\,s^{\alpha-1}}$) depends on α ([1]).

1.2 Circuit (R+Q)

1.2.1 Impedance

$$Z(\omega) = R + \frac{1}{Q\,(\mathrm{i}\,\omega)^{\alpha}}\ ,\ \mathrm{Re}\,Z = R + \frac{c_\alpha}{Q\,\omega^{\alpha}}\ ,\ \mathrm{Im}\,Z = -\frac{s_\alpha}{Q\,\omega^{\alpha}}$$

[1] Different equations for CPE: $Z = \frac{Q}{(\mathrm{i}\,\omega)^{1-\alpha}}$ [6], $Z = \frac{1}{(Q\,\mathrm{i}\,\omega)^{\alpha}}$ [33].

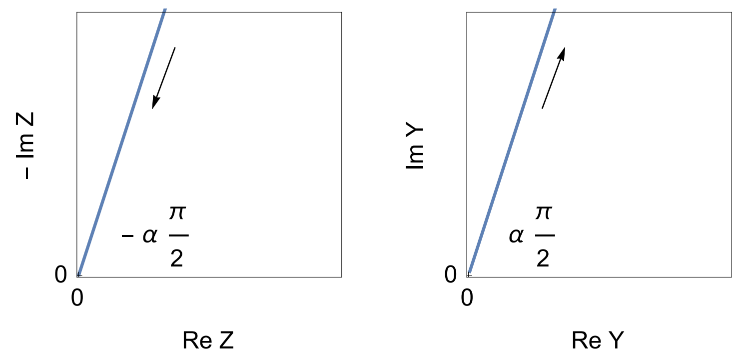

Figure 1.2: Nyquist diagram of the impedance and admittance for the CPE element, plotted for $\alpha = 0.8$. The arrows always indicate the increasing frequency direction.

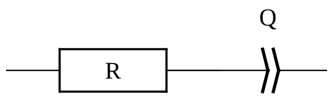

Figure 1.3: Circuit (R+Q).

1.2.2 Reduced impedance

$$Z^*(\omega) = \frac{Z(\omega)}{R} = 1 + \frac{1}{\tau\,(\mathrm{i}\,\omega)^{\alpha}}\ ,\ \tau = R\,Q$$

The τ unit depends on α: $\mathrm{u}_\tau = \mathrm{s}^\alpha$.

$$Z^*(u) = 1 + \frac{1}{(\mathrm{i}\,u)^{\alpha}},\ u = \omega\,\tau^{1/\alpha}$$

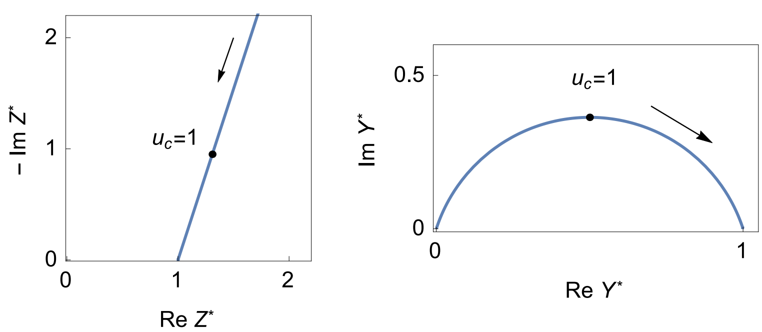

Figure 1.4: Nyquist diagram of the reduced impedance and admittance ($Y^* = R\,Y$) for the (R+Q) circuit, plotted for $\alpha = 0.8$.

1.3 Circuit (R/Q)

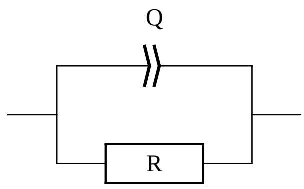

Figure 1.5: Circuit (R/Q).

1.3.1 Impedance

$$Z(\omega) = \frac{R}{1+\tau\,(\mathrm{i}\,\omega)^{\alpha}} \;;\; \tau = R\,Q$$

$$\mathrm{Re}\ Z(\omega) = \frac{R\,(1+\tau\,\omega^{\alpha}\,c_{\alpha})}{1+\tau^2\,\omega^{2\,\alpha}+2\,\tau\,\omega^{\alpha}\,c_{\alpha}} \;;\; \mathrm{Im}\ Z(\omega) = -\frac{R\,\tau\,\omega^{\alpha}\,s_{\alpha}}{1+\tau^2\,\omega^{2\,\alpha}+2\,\tau\,\omega^{\alpha}\,c_{\alpha}}$$

1.3.2 Reduced impedance

$$Z^*(\omega) = \frac{Z(\omega)}{R} = \frac{1}{1+\tau\,(\mathrm{i}\,\omega)^{\alpha}} \;;\; \tau = R\,Q$$

$$\mathrm{Re}\ Z^*(\omega) = \frac{1+\tau\,\omega^{\alpha}\,c_{\alpha}}{1+\tau^2\,\omega^{2\,\alpha}+2\,\tau\,\omega^{\alpha}\,c_{\alpha}} \;;\; \mathrm{Im}\ Z^*(\omega) = -\frac{\tau\,\omega^{\alpha}\,s_{\alpha}}{1+\tau^2\,\omega^{2\,\alpha}+2\,\tau\,\omega^{\alpha}\,c_{\alpha}}$$

$$\frac{\mathrm{dIm}\ Z^*(\omega)}{\mathrm{d}\omega} = \frac{\alpha\,\tau\,\omega^{-1+\alpha}\,\left(-1+\tau^2\,\omega^{2\,\alpha}\right)\,s_{\alpha}}{\left(1+\tau^2\,\omega^{2\,\alpha}+2\,\tau\,\omega^{\alpha}\,c_{\alpha}\right)^2} = 0 \Rightarrow \omega_c^{\alpha} = 1/\tau \text{ [7]}$$

$$\mathrm{Re}\ Z^*(\omega_c) = 1/2\ ,\ \mathrm{Im}\ Z^*(\omega_c) = -\frac{s_{\alpha}}{2\,(1+c_{\alpha})}$$

$$\alpha = \frac{2}{\pi}\ \arccos\left(-1+\frac{2}{1+4\,\mathrm{Im}\ Z^*(\omega_c)^2}\right)$$

$$Z^*(u) = \frac{1}{1+(\mathrm{i}\,u)^{\alpha}},\ u = \omega\,\tau^{1/\alpha}$$

(Figs. 1.6, 1.7)

1.3.3 Pseudocapacitance #1

The value of the pseudocapacitance C ($C/(\mathrm{F\ cm^{-2}})$) for the (R/C) circuit giving the same characteristic frequency than that of the (R/Q) circuit (Fig. 1.8) is obtained from [6]:

$$\omega_{\mathrm{c}} = \frac{1}{(R\,Q)^{1/\alpha}} = \frac{1}{R\,C} \Rightarrow C = Q^{1/\alpha}R^{(1-\alpha)/\alpha}$$

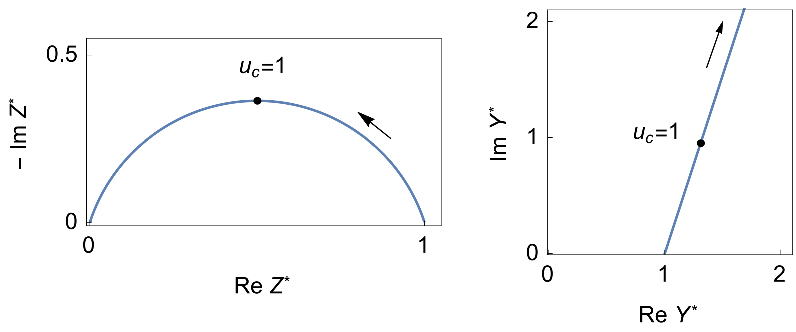

Figure 1.6: Nyquist diagram of the reduced impedance (depressed semi-circle [28]) and admittance ($Y^* = R\,Y$) for the (R/Q) circuit, plotted for $\alpha = 0.8$.

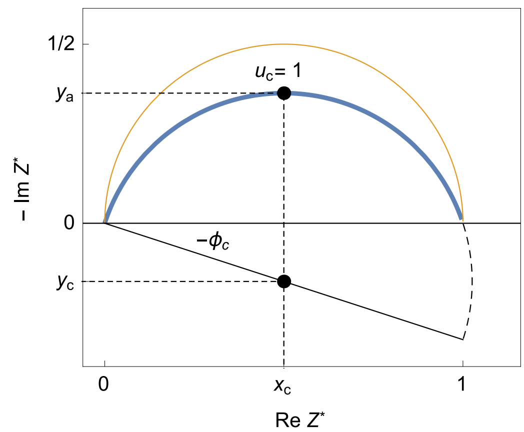

Figure 1.7: The Nyquist diagram of the reduced impedance belongs to the circle with $x_c = 1/2$, $y_c = -c_\alpha/(2\,s_\alpha)$ and radius $r = 1/(2\,s_\alpha)$. $\phi_c = (1-\alpha)\,\pi/2$ [2].

1.3.4 Pseudocapacitance #2

The value of the pseudocapacitance C ($C/(\mathrm{F\ cm^{-2}})$) for the ($\mathrm{R_C}$/C) circuit giving the same impedance for the characteristic frequency of the ($\mathrm{R_Q}$/Q) circuit (Fig. 1.8) is obtained from [3, 11]:

$$C = Q^{1/\alpha} R_{\mathrm{Q}}^{(1/\alpha)-1}\, \sin(\alpha\,\pi/2),\ R_{\mathrm{C}} = \frac{R_{\mathrm{Q}}}{2\,(\cos(\alpha\,\pi/4))^2}$$

with:

$$\tau_{(\mathrm{R_C/C})} = (R_{\mathrm{Q}}\,Q)^{1/\alpha}\tan(\alpha\,\pi/4)$$

1.4 Circuit (R/Q)+(R/Q)+ .. (Voigt)

$$Z(\omega) = \sum_{i=1}^{n_{\mathrm{RQ}}} \frac{R_i}{1+\tau_i\,(\mathrm{i}\,\omega)^{\alpha_i}}\ ;\ \tau_i = R_i\,Q_i$$

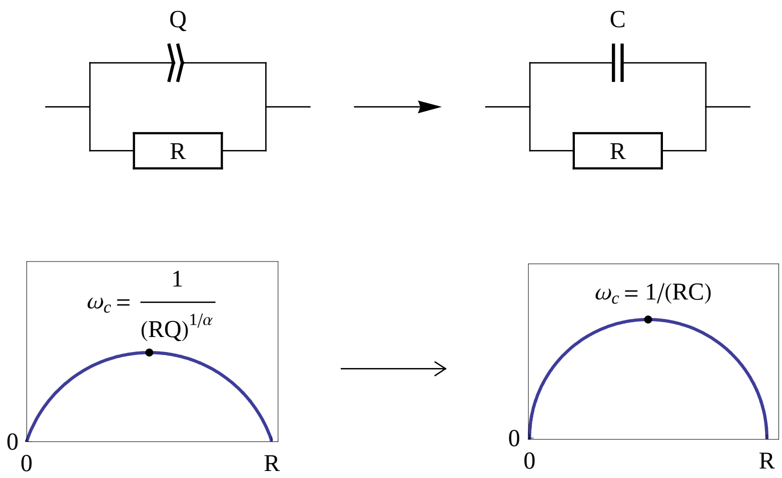

Figure 1.8: (R/Q) and (R/C) circuits with the same characteristic frequency at the apex (or summit) of impedance arc.

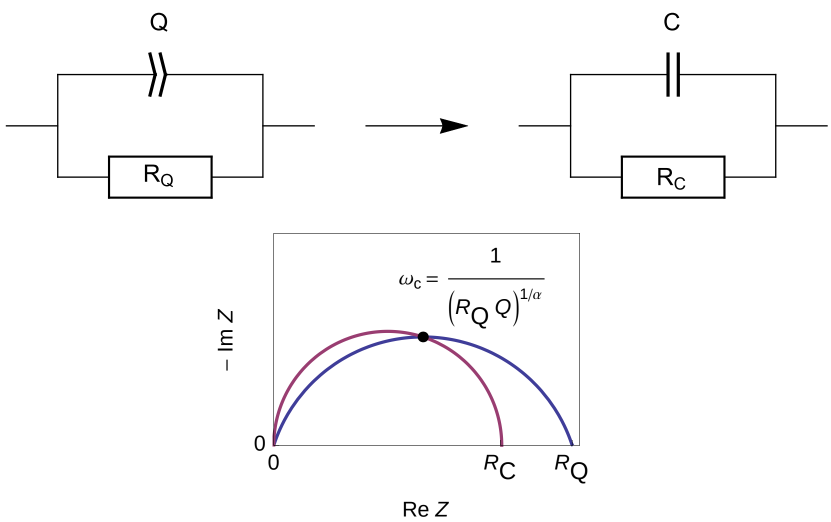

Figure 1.9: (R_Q/Q) and (R_C/C) circuits with the same impedance for the characteristic frequency of the (R_Q/Q) circuit.

$$\mathrm{Re}\ Z(\omega) = \sum_{i=1}^{n_{\mathrm{RQ}}} \frac{R_i\ (1 + \tau_i\, \omega^{\alpha_i}\, c_{\alpha i})}{1 + \tau_i^2\, \omega^{2\,\alpha_i} + 2\,\tau_i\, \omega^{\alpha_i}\, c_{\alpha i})}$$

$$\mathrm{Im}\, Z(\omega) = -\sum_{i=1}^{n_{\mathrm{RQ}}} \frac{R_i\, \tau_i\, \omega^{\alpha_i}\, s_{\alpha i}}{1 + \tau_i^2\, \omega^{2\,\alpha_i} + 2\, \tau_i\, \omega^{\alpha_i}\, c_{\alpha i}}$$

1.5 Circuit ($\mathbf{R_1}$+($\mathbf{R_2}$/$\mathbf{Q_2}$))

Fig. 1.10.

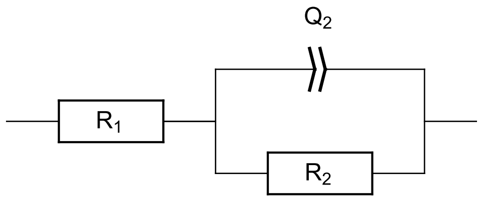

Figure 1.10: Circuit (R_1+(R_2/Q_2)).

1.5.1 Impedance

$$Z(\omega) = R_1 + \frac{1}{(\mathrm{i}\,\omega)^{\alpha_2}\, Q_2 + \dfrac{1}{R_2}}$$

$$Z(\omega) = \frac{(R_1 + R_2)\ (1 + (\mathrm{i}\,\omega)^{\alpha_2}\, \tau_2)}{1 + (\mathrm{i}\,\omega)^{\alpha_2}\, \tau_1},\ \tau_1 = R_2\, Q_2,\ \tau_2 = \frac{R_1\, R_2\, Q_2}{R_1 + R_2}$$

1.5.2 Reduced impedance

$$Z^*(u) = \frac{Z(u)}{R_1 + R_2} = \frac{1 + T\,(\mathrm{i}\,u)^{\alpha_2}}{1 + (\mathrm{i}\,u)^{\alpha_2}} \tag{1.1}$$

$$u = \tau_1^{1/\alpha_2}\, \omega,\ T = \tau_2/\tau_1 = R_1/(R_1 + R_2) < 1$$

$$\mathrm{Re}\, Z^*(u) = \frac{T\, c_\alpha\, u^{\alpha_2} + c_\alpha\, u^{\alpha_2} + T u^{2\alpha_2} + 1}{2\, c_\alpha\, u^{\alpha_2} + u^{2\alpha_2} + 1}$$

$$\mathrm{Im}\, Z^*(u) = -\frac{(1 - T)\, u^{\alpha_2}\, s_\alpha}{2\, c_\alpha\, u^{\alpha_2} + u^{2\alpha_2} + 1}$$

1.6 Circuit ($\mathbf{R_1}$/($\mathbf{R_2}$+$\mathbf{Q_2}$))

Fig. 1.12.

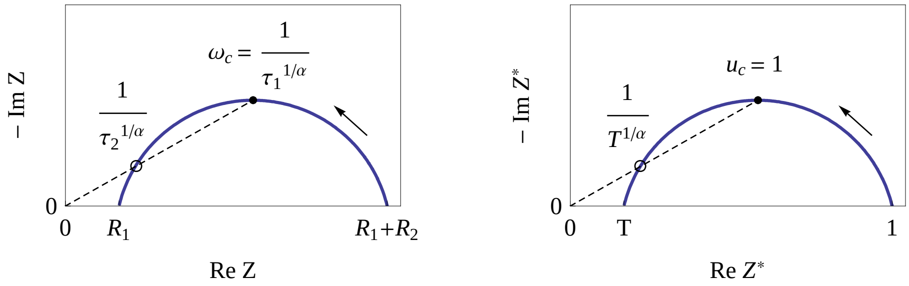

Figure 1.11: Nyquist diagrams of the impedance and reduced impedance for the (R_1+(R_2/Q_2)) circuit.

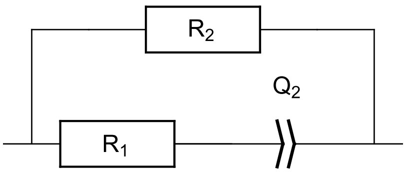

Figure 1.12: Circuit (R_1/(R_2+Q_2)).

1.6.1 Impedance

$$Z(\omega) = \frac{R_1\,(1 + \tau_2\,(\mathrm{i}\,\omega)^{\alpha_2})}{1 + \tau_1\,(\mathrm{i}\,\omega)^{\alpha_2}},\ \tau_1 = (R_1 + R_2)\,Q_2,\ \tau_2 = R_2\,Q_2$$

$$\mathrm{Re}\ Z(\omega) = \frac{R_1\left(\cos\left(\frac{\pi\alpha_2}{2}\right)(\tau_1 + \tau_2)\,\omega^{\alpha_2} + \tau_1\tau_2\omega^{2\alpha_2} + 1\right)}{\tau_1\left(\tau_1\omega^{\alpha_2} + 2\cos\left(\frac{\pi\alpha_2}{2}\right)\right)\omega^{\alpha_2} + 1}$$

$$\mathrm{Im}\ Z(\omega) = -\frac{\omega^{\alpha_2}\sin\left(\frac{\pi\alpha_2}{2}\right)R_1\,(\tau_1 - \tau_2)}{\tau_1\left(\tau_1\omega^{\alpha_2} + 2\cos\left(\frac{\pi\alpha_2}{2}\right)\right)\omega^{\alpha_2} + 1}$$

1.6.2 Reduced impedance

$$Z^*(u) = \frac{Z(u)}{R_1} = \frac{1 + T\,(\mathrm{i}\,u)^{\alpha_2}}{1 + (\mathrm{i}\,u)^{\alpha_2}}$$

$$u = \tau_1^{1/\alpha_2}\,\omega,\ T = \tau_2/\tau_1 = R_2/(R_1 + R_2) < 1$$

cf. Eq. (1.1) and Fig. 1.11.

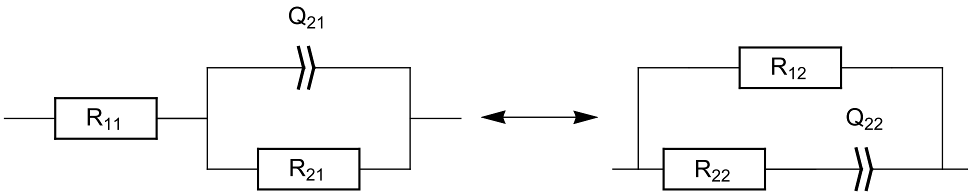

Figure 1.13: The (R+(R/Q)) and (R/(R+Q)) circuits are non-distinguishable for $\alpha_{21} = \alpha_{22}$ [1].

1.7 Transformation formulae between(R+(R/Q)) and (R/(R+Q))

1.7.1 $\alpha_{21} = \alpha_{22}$

Transformations formulae (R+(R/Q)) → (R/(R+Q))

$$R_{12} = R_{11} + R_{21}, R_{22} = \frac{R_{11}^2}{R_{21}} + R_{11}, Q_{22} = \frac{Q_{21} R_{21}^2}{(R_{11} + R_{21})^2}$$

Transformations formulae (R/(R+Q))→(R+(R/Q))

$$Q_{21} = \frac{Q_{22}(R_{12} + R_{22})^2}{R_{12}^2}, R_{11} = \frac{R_{12} R_{22}}{R_{12} + R_{22}}, R_{21} = \frac{R_{12}^2}{R_{12} + R_{22}}$$

1.7.2 $\alpha_{21} \neq \alpha_{22}$

The (R+(R/Q)) and (R/(R+Q)) circuits (Fig. 1.13) are distinguishable for $\alpha_{21} \neq \alpha_{22}$

Chapter 2

Circuits made of two CPEs

2.1 Circuit (Q_1+Q_2)

Fig. 2.1.

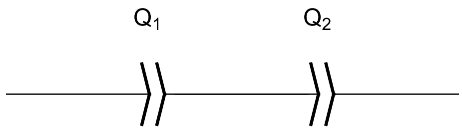

Figure 2.1: Circuit (Q_1+Q_2).

2.1.1 $\alpha_1 = \alpha_2 = \alpha$

$$Z(\omega) = \left(\frac{1}{Q_1} + \frac{1}{Q_2}\right) \frac{1}{(\mathrm{i}\,\omega)^{\alpha}} = \frac{1}{Q\,(\mathrm{i}\,\omega)^{\alpha}},\; Q = \frac{Q_1\,Q_2}{Q_1 + Q_2}$$

cf. § 1.1.

2.1.2 $\alpha_1 \neq \alpha_2$

Impedance

$$Z(\omega) = \frac{1}{Q_1\,(\mathrm{i}\,\omega)^{\alpha_1}} + \frac{1}{Q_2\,(\mathrm{i}\,\omega)^{\alpha_2}} = \frac{Q_1\,(\mathrm{i}\,\omega)^{\alpha_1} + Q_2\,(\mathrm{i}\,\omega)^{\alpha_2}}{Q_1\,Q_2\,(\mathrm{i}\,\omega)^{\alpha_1+\alpha_2}}$$

$$\mathrm{Re}\;Z(\omega) = \frac{\cos\left(\frac{\pi\alpha_1}{2}\right)\omega^{-\alpha_1}}{Q_1} + \frac{\cos\left(\frac{\pi\alpha_2}{2}\right)\omega^{-\alpha_2}}{Q_2}$$

$$\mathrm{Im}\;Z(\omega) = -\frac{\sin\left(\frac{\pi\alpha_1}{2}\right)\omega^{-\alpha_1}}{Q_1} - \frac{\sin\left(\frac{\pi\alpha_2}{2}\right)\omega^{-\alpha_2}}{Q_2}$$

$$|Z_{\mathrm{Q}_1}| = |Z_{\mathrm{Q}_2}| \Rightarrow \omega = \omega_{\mathrm{c}} = \left(\frac{Q_2}{Q_1}\right)^{\frac{1}{\alpha_1-\alpha_2}}$$

- $\alpha_1 < \alpha_2$ (Figs. 2.2 and 2.3)

$$\omega \to 0 \Rightarrow Z(\omega) \approx \frac{1}{Q_2\,(\mathrm{i}\,\omega)^{\alpha_2}},\; \omega \to \infty \Rightarrow Z(\omega) \approx \frac{1}{Q_1\,(\mathrm{i}\,\omega)^{\alpha_1}}$$

- $\alpha_1 > \alpha_2$

$$\omega \to 0 \Rightarrow Z(\omega) \approx \frac{1}{Q_1\,(\mathrm{i}\,\omega)^{\alpha_1}},\; \omega \to \infty \Rightarrow Z(\omega) \approx \frac{1}{Q_2\,(\mathrm{i}\,\omega)^{\alpha_2}}$$

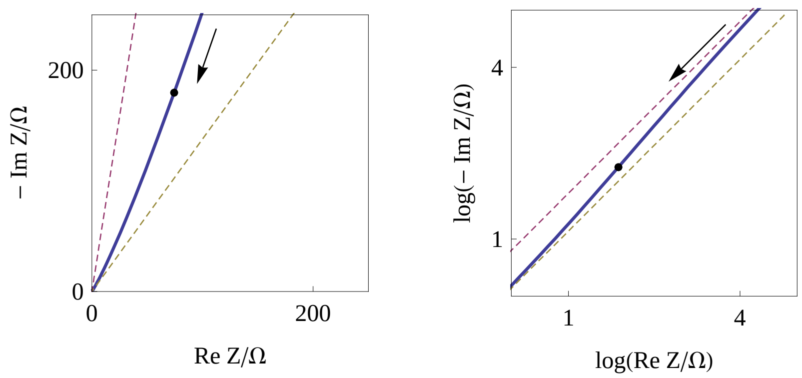

Figure 2.2: Nyquist and log Nyquist [10] diagrams of the impedance for the (Q_1+Q_2) circuit, plotted for $Q_1 = 10^{-2}$ F cm^{-2} s$^{\alpha_1-1}$, $Q_2 = 10^{-2}$ F cm^{-2} s$^{\alpha_2-1}$, $\alpha_1 = 0.6, \alpha_2 = 0.9$ ($\alpha_1 < \alpha_2$). Dots: $\omega_c = (Q_2/Q_1)^{1/(\alpha 1-\alpha_2)}$.

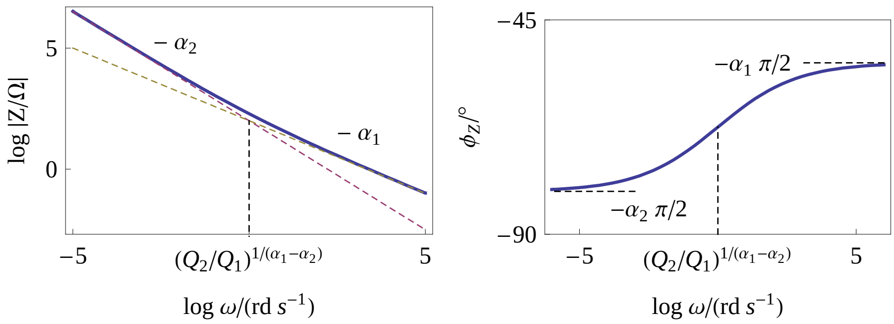

Figure 2.3: Bode diagrams of the impedance for the (Q_1+Q_2) circuit. Same values of parameters as in Fig. 2.2. $\alpha_1 < \alpha_2$.

2.1.3 Reduced impedance

$$Z^*(u) = Q_1\,\omega_c^{\alpha_1}\,Z(\omega) = \frac{1}{(\mathrm{i}\,u)^{\alpha_1}} + \frac{1}{(\mathrm{i}\,u)^{\alpha_2}},\; u = \frac{\omega}{\omega_c}$$

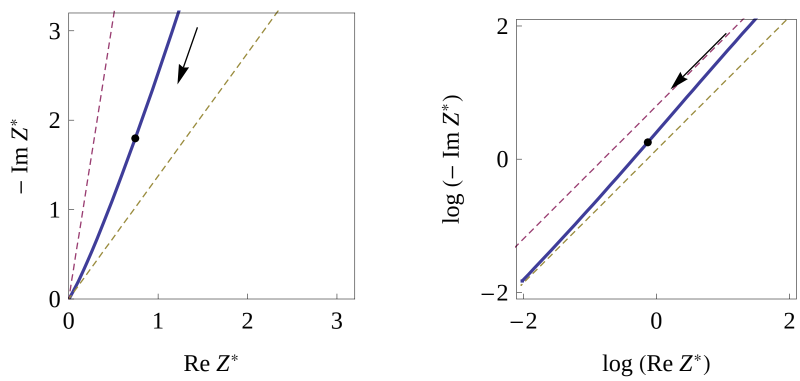

Figure 2.4: Nyquist and log Nyquist [10] diagrams of the reduced impedance for the (Q_1+Q_2) circuit, plotted for $\alpha_1 = 0.6, \alpha_2 = 0.9$ ($\alpha_1 < \alpha_2$). Dots: $u_c = 1$.

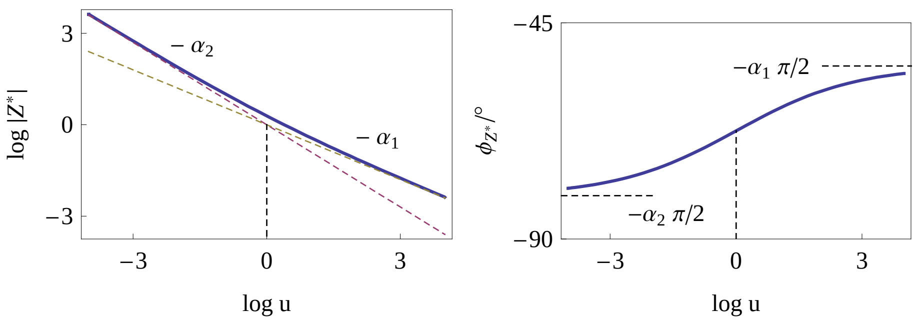

Figure 2.5: Bode diagrams of the impedance for the (Q_1+Q_2) circuit. Same values of parameters as in Fig. 2.4. $\alpha_1 < \alpha_2$.

2.2 Circuit ($\mathrm{Q}_1/\mathrm{Q}_2$)

Fig. 2.6

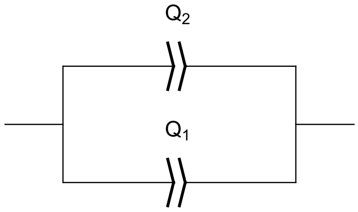

Figure 2.6: Circuit ($\mathrm{Q}_1/\mathrm{Q}_2$).

2.2.1 $\alpha_1 = \alpha_2 = \alpha$

$$Z(\omega) = \frac{1}{(Q_1 + Q_2)\,(\mathrm{i}\,\omega)^{\alpha}} = \frac{1}{Q\,(\mathrm{i}\,\omega)^{\alpha}},\ Q = Q_1 + Q_2$$

cf. § 1.1.

2.2.2 $\alpha_1 \neq \alpha_2$

Impedance

$$Z(\omega) = \frac{1}{Q_1\,(\mathrm{i}\,\omega)^{\alpha_1} + Q_2\,(\mathrm{i}\,\omega)^{\alpha_2}}$$

$$\mathrm{Re}\ Z(\omega) = \frac{\cos\left(\frac{\pi\alpha_1}{2}\right) Q_1\omega^{\alpha_1} + \cos\left(\frac{\pi\alpha_2}{2}\right) Q_2\omega^{\alpha_2}}{Q_1^2\omega^{2\alpha_1} + Q_2^2\omega^{2\alpha_2} + 2\cos\left(\frac{1}{2}\pi\,(\alpha_1 - \alpha_2)\right) Q_1 Q_2\omega^{\alpha_1+\alpha_2}}$$

$$\mathrm{Im}\ Z(\omega) = -\frac{\sin\left(\frac{\pi\alpha_1}{2}\right) Q_1\omega^{\alpha_1} + \sin\left(\frac{\pi\alpha_2}{2}\right) Q_2\omega^{\alpha_2}}{Q_1^2\omega^{2\alpha_1} + Q_2^2\omega^{2\alpha_2} + 2\cos\left(\frac{1}{2}\pi\,(\alpha_1 - \alpha_2)\right) Q_1 Q_2\omega^{\alpha_1+\alpha_2}}$$

- $\alpha_1 < \alpha_2$ (Figs. 2.7 and 2.8)

$$\omega \to 0 \Rightarrow Z(\omega) \approx \frac{1}{Q_1\,(\mathrm{i}\,\omega)^{\alpha_1}},\ \omega \to \infty \Rightarrow Z(\omega) \approx \frac{1}{Q_2\,(\mathrm{i}\,\omega)^{\alpha_2}}$$

- $\alpha_1 > \alpha_2$

$$\omega \to 0 \Rightarrow Z(\omega) \approx \frac{1}{Q_2\,(\mathrm{i}\,\omega)^{\alpha_2}},\ \omega \to \infty \Rightarrow Z(\omega) \approx \frac{1}{Q_1\,(\mathrm{i}\,\omega)^{\alpha_1}}$$

2.2.3 Reduced impedance

$$Z^*(u) = Q_1\,\omega_{\mathrm{c}}^{\alpha_1}\,Z(\omega) = \frac{1}{(\mathrm{i}\,u)^{\alpha_1} + (\mathrm{i}\,u)^{\alpha_2}},\ u = \frac{\omega}{\omega_{\mathrm{c}}}$$

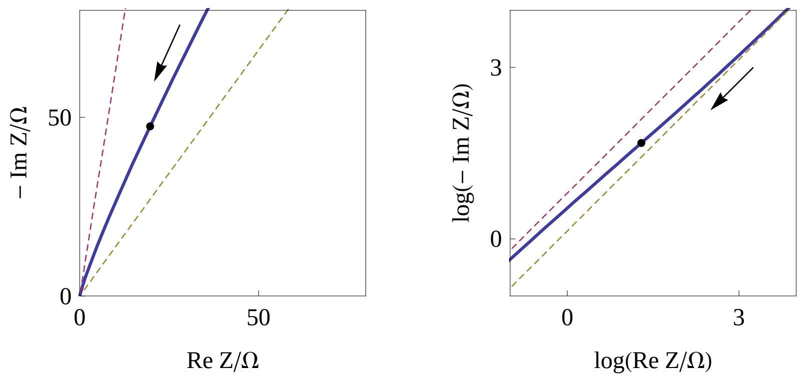

Figure 2.7: Nyquist and log Nyquist [10] diagrams of the impedance for the ($\mathrm{Q}_1/\mathrm{Q}_2$) circuit plotted for $Q_1 = 10^{-2}$ F cm^{-2} s$^{\alpha_1-1}$, $Q_2 = 10^{-2}$ F cm^{-2} s$^{\alpha_2-1}$, $\alpha_1 = 0.6, \alpha_2 = 0.9$ ($\alpha_1 < \alpha_2$). Dots: $\omega_c = (Q_2/Q_1)^{1/(\alpha 1-\alpha_2)}$.

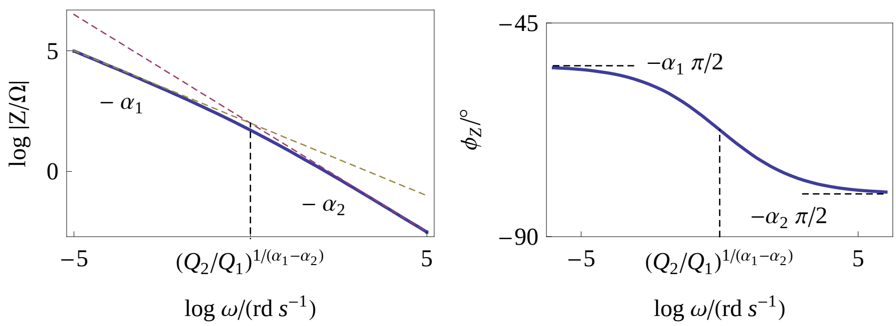

Figure 2.8: Bode diagrams of the impedance for the ($\mathrm{Q}_1/\mathrm{Q}_2$) circuit. Same values of parameters as in Fig. 2.7. $\alpha_1 < \alpha_2$.

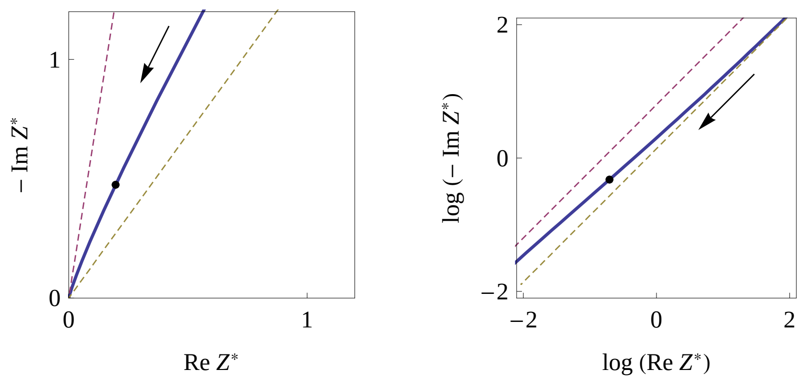

Figure 2.9: Nyquist and log Nyquist [10] diagrams of the reduced impedance for the ($\mathrm{Q}_1/\mathrm{Q}_2$) circuit, plotted for $\alpha_1 = 0.6, \alpha_2 = 0.9$ ($\alpha_1 < \alpha_2$). Dots: $u_c = 1$.

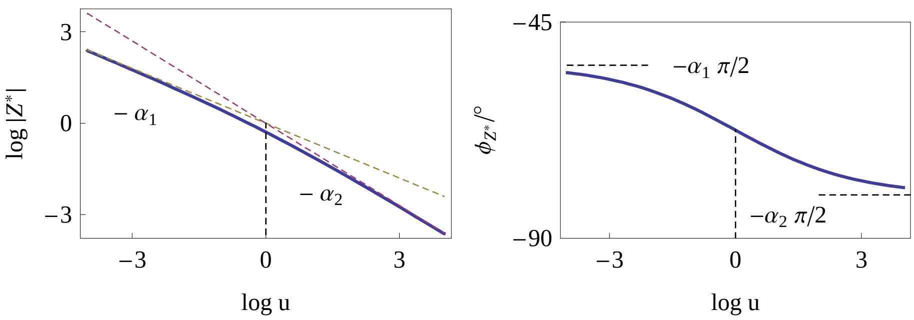

Figure 2.10: Bode diagrams of the impedance for the (Q_1/Q_2) circuit. Same values of parameters as in Fig. 2.9. $\alpha_1 < \alpha_2$.

Chapter 3

Circuits made of one R and two CPEs

3.1 Circuit $((\mathbf{R}_1/\mathbf{Q}_1) + \mathbf{Q}_2)$

Fig. 3.1.

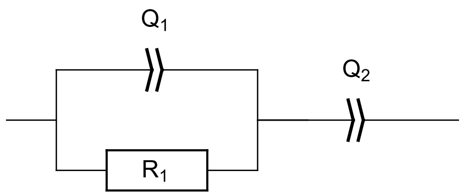

Figure 3.1: Circuit $((\mathrm{R}_1/\mathrm{Q}_1)+\mathrm{Q}_2)$.

3.1.1 $\alpha_1 = \alpha_2 = \alpha$

Impedance

$$Z(\omega) = \frac{1}{\dfrac{1}{R_1} + Q_1\,(\mathrm{i}\,\omega)^\alpha} + \frac{1}{Q_2\,(\mathrm{i}\,\omega)^\alpha}$$

$$Z(\omega) = \frac{1 + (\mathrm{i}\,\omega)^\alpha\,\tau_2}{(\mathrm{i}\,\omega)^\alpha\,Q_2\,(1 + (\mathrm{i}\,\omega)^\alpha\,\tau_1)},\ \tau_1 = R_1\,Q_1,\ \tau_2 = (Q_1 + Q_2)\,R_1,\ \tau_1 < \tau_2$$

$$\mathrm{Re}\ Z(\omega) = -\frac{\cos\left(\frac{\pi\alpha}{2}\right)\left(\tau_1\tau_2\omega^{2\alpha}+1\right)\omega^{-\alpha} + \cos(\pi\alpha)\tau_1 + \tau_2}{\mathrm{Q2}\left(\tau_1\left(\tau_1\omega^\alpha + 2\cos\left(\frac{\pi\alpha}{2}\right)\right)\omega^\alpha + 1\right)}$$

$$\mathrm{Im}\ Z(\omega) = -\frac{\sin\left(\frac{\pi\alpha}{2}\right)\left(\tau_1\tau_2\omega^{2\alpha}+1\right)\omega^{-\alpha} + \sin(\pi\alpha)\tau_1}{\mathrm{Q2}\left(\tau_1\left(\tau_1\omega^\alpha + 2\cos\left(\frac{\pi\alpha}{2}\right)\right)\omega^\alpha + 1\right)}$$

Reduced impedance

$$Z^*(u) = \frac{Z(u)}{R_1} = \frac{1}{T-1}\,\frac{1+T\,(\mathrm{i}\,u)^\alpha}{(\mathrm{i}\,u)^\alpha\,(1+(\mathrm{i}\,u)^\alpha)} \tag{3.1}$$

$$u = \omega\,\tau^{1/\alpha},\ T = \tau_2/\tau_1 = 1 + Q_2/Q_1 > 1$$

$$\mathrm{Re}\ Z^*(u) = \frac{u^{-\alpha}\left((T+\cos(\alpha\pi))u^\alpha + \left(Tu^{2\alpha}+1\right)\cos\left(\frac{\alpha\pi}{2}\right)\right)}{(T-1)\left(2\cos\left(\frac{\alpha\pi}{2}\right)u^\alpha + u^{2\alpha}+1\right)}$$

$$\mathrm{Im}\ Z^*(u) = u^{-\alpha}\left(\frac{1}{1-T} - \frac{u^{2\alpha}}{2\cos\left(\frac{\alpha\pi}{2}\right)u^\alpha + u^{2\alpha}+1}\right)\sin\left(\frac{\alpha\pi}{2}\right)$$

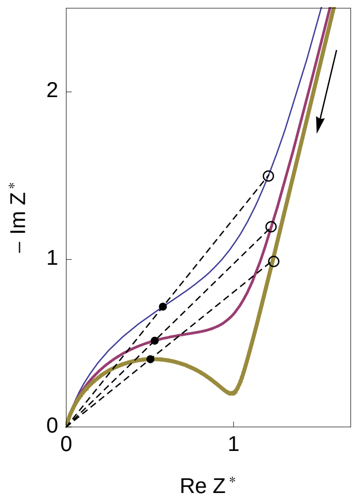

Figure 3.2: Nyquist diagram of the reduced impedance for the $((\mathrm{R}_1/\mathrm{Q}_1)+\mathrm{Q}_2)$ circuit (Fig. 3.1, Eq. (3.1)), plotted for $T = 4, 9, 90$ and $\alpha = 0.85$. The line thickness increases with increasing T. Dots: reduced characteristic angular frequency $u_{c1} = 1$; circles: reduced characteristic angular frequency $u_{c2} = 1/T^{1/\alpha}$ $(\phi_{u_{c1}} = \phi_{u_{c2}})$.

3.1.2 $\alpha_1 \neq \alpha_2$

Impedance

$$Z(\omega) = \frac{1}{\dfrac{1}{R_1} + Q_1\,(\mathrm{i}\,\omega)^{\alpha_1}} + \frac{1}{Q_2\,(\mathrm{i}\,\omega)^{\alpha_2}}$$

$$\mathrm{Re}\ Z(\omega) = \frac{\cos\left(\frac{\pi\alpha_2}{2}\right)\omega^{-\alpha_2}}{Q_2} + \frac{R_1\left(\cos\left(\frac{\pi\alpha_1}{2}\right)Q_1R_1\omega^{\alpha_1}+1\right)}{Q_1R_1\left(Q_1R_1\omega^{\alpha_1}+2\cos\left(\frac{\pi\alpha_1}{2}\right)\right)\omega^{\alpha_1}+1}$$

$$\mathrm{Im}\ Z(\omega) = -\frac{\sin\left(\frac{\pi\alpha_1}{2}\right)Q_1R_1^2\omega^{\alpha_1}}{Q_1R_1\left(Q_1R_1\omega^{\alpha_1}+2\cos\left(\frac{\pi\alpha_1}{2}\right)\right)\omega^{\alpha_1}+1} - \frac{\sin\left(\frac{\pi\alpha_2}{2}\right)\omega^{-\alpha_2}}{Q_2}$$

3.2 Circuit $((\mathbf{R}_1 + \mathbf{Q}_1)/\mathbf{Q}_2)$

Fig. 3.3

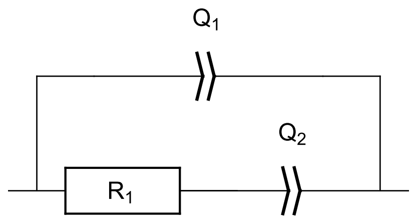

Figure 3.3: Circuit ((R1+Q2)/Q1).

3.2.1 $\alpha_1 = \alpha_2 = \alpha$

Impedance

$$Z(\omega) = \frac{1}{(\mathrm{i}\,\omega)^\alpha\, Q_1 + \dfrac{1}{R_1 + \dfrac{1}{(\mathrm{i}\,\omega)^\alpha\, Q_2}}} = \frac{1 + Q_2\, R_1 (\mathrm{i}\;\omega)^\alpha}{(\mathrm{i}\;\omega)^\alpha\,(Q_1 + Q_2)\left(1 + \dfrac{(\mathrm{i}\;\omega)^\alpha\, Q_1\, Q_2\, R_1}{Q_1 + Q_2}\right)}$$

$$Z(\omega) = \frac{1 + \tau_2 (\mathrm{i}\;\omega)^\alpha}{(\mathrm{i}\;\omega)^\alpha\,(Q_1 + Q_2)\;(1 + (\mathrm{i}\;\omega)^\alpha\,\tau_1)},\ \tau_1 = \frac{Q_1\, Q_2\, R_1}{Q_1 + Q_2},\ \tau_2 = Q_2\, R_1$$

$$\mathrm{Re}\; Z(\omega) = \frac{\omega^{-\alpha}\left(\cos(\pi\alpha)\omega^\alpha + \tau_2\omega^\alpha + \cos\left(\frac{\pi\alpha}{2}\right)\left(\tau_2\omega^{2\alpha} + 1\right)\right)}{\left(2\cos\left(\frac{\pi\alpha}{2}\right)\omega^\alpha + \omega^{2\alpha} + 1\right)(Q_1 + Q_2)\,\tau_1}$$

$$\mathrm{Im}\; Z(\omega) = -\frac{\omega^{-\alpha}\sin\left(\frac{\pi\alpha}{2}\right)\left(2\cos\left(\frac{\pi\alpha}{2}\right)\omega^\alpha + \tau_2\omega^{2\alpha} + 1\right)}{\left(2\cos\left(\frac{\pi\alpha}{2}\right)\omega^\alpha + \omega^{2\alpha} + 1\right)(Q_1 + Q_2)\,\tau_1}$$

Reduced impedance

$$Z^*(u) = \frac{Z(u)}{R_1} = \frac{T-1}{T^2}\,\frac{1 + T\,(\mathrm{i}\,u)^\alpha}{(\mathrm{i}\,u)^\alpha\,(1 + (\mathrm{i}\,u)^\alpha)} \tag{3.2}$$

$$u = \omega\,\tau^{1/\alpha},\ T = \tau_2/\tau_1 = 1 + Q_2/Q_1 > 1$$

$$\mathrm{Re}\; Z^*(u) = \frac{(T-1)u^{-\alpha}\left((T + \cos(\alpha\pi))u^\alpha + \left(Tu^{2\alpha} + 1\right)\cos\left(\frac{\alpha\pi}{2}\right)\right)}{T^2\left(2\cos\left(\frac{\alpha\pi}{2}\right)u^\alpha + u^{2\alpha} + 1\right)}$$

$$\mathrm{Im}\; Z^*(u) = -\frac{(T-1)u^{-\alpha}\left(2\cos\left(\frac{\alpha\pi}{2}\right)u^\alpha + Tu^{2\alpha} + 1\right)\sin\left(\frac{\alpha\pi}{2}\right)}{T^2\left(2\cos\left(\frac{\alpha\pi}{2}\right)u^\alpha + u^{2\alpha} + 1\right)}$$

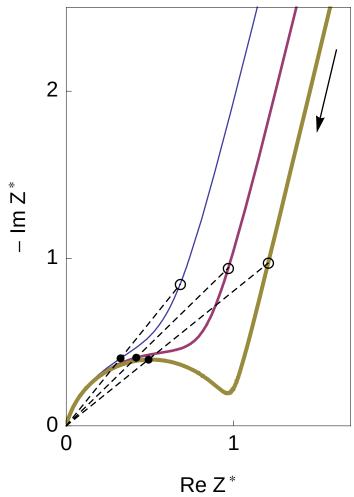

Figure 3.4: Nyquist diagram of the reduced impedance for the ((R$_1$+Q$_1$)/Q$_2$) circuit (Fig. 3.3, Eq. (3.2)), plotted for $T = 4, 9, 90$ and $\alpha = 0.85$. The line thickness increases with increasing T. Dots: reduced characteristic angular frequency $u_{c1} = 1$; circles: reduced characteristic angular frequency $u_{c2} = 1/T^{1/\alpha}$ ($\phi_{u_{c1}} = \phi_{u_{c2}}$).

3.2.2 $\alpha_1 \neq \alpha_2$

$$Z(\omega) = \frac{\dfrac{1}{(\mathrm{i}\,\omega)^{\alpha_2}\,Q_2} + R_1}{(\mathrm{i}\,\omega)^{\alpha_1}\,Q_1\left(\dfrac{1}{(\mathrm{i}\,\omega)^{\alpha_1}\,Q_1} + \dfrac{1}{(\mathrm{i}\,\omega)^{\alpha_2}\,Q_2} + R_1\right)}$$

$$Z(\omega) = \frac{1 + \tau\,(\mathrm{i}\,\omega)^{\alpha_2}}{(\mathrm{i}\,\omega)^{\alpha_1}\;Q_1 + (\mathrm{i}\,\omega)^{\alpha_2}\;Q_2 + \tau\,(\mathrm{i}\,\omega)^{\alpha_1+\alpha_2}\,Q_1}\;,\;\tau = R_1\,Q_2$$

$$\mathrm{Re}\;Z(\omega) =$$
$$\left(\omega^{\alpha_1}\,c_{\alpha 1}\,\left(1 + \tau^2\,\omega^{2\,\alpha_2} + 2\,\tau\,\omega^{\alpha_2}\,c_{\alpha 2}\right)\,Q_1 + \omega^{\alpha_2}\,\left(\tau\,\omega^{\alpha_2} + c_{\alpha 2}\right)\,Q_2\right)/$$
$$\left(\omega^{2\,\alpha_1}\,\left(1 + \tau^2\,\omega^{2\,\alpha_2} + 2\,\tau\,\omega^{\alpha_2}\,c_{\alpha 2}\right)\,{Q_1}^2 + 2\,\omega^{\alpha_1+\alpha_2}\,\left(\tau\,\omega^{\alpha_2}\,c_{\alpha 1} + c_{\alpha 1 m \alpha 2}\right)\,Q_1\,Q_2 + \omega^{2\,\alpha_2}\,{Q_2}^2\right)$$

$$c_{\alpha 1 m \alpha 2} = \cos\left(\frac{\pi\;(\alpha_1 - \alpha_2)}{2}\right)$$

$$\mathrm{Im}\;Z(\omega) =$$
$$\left(-\omega^{\alpha_1}\,\left(1 + \tau^2\,\omega^{2\,\alpha_2} + 2\,\tau\,\omega^{\alpha_2}\,c_{\alpha 2}\right)\,Q_1\,s_{\alpha 1} - \omega^{\alpha_2}\,Q_2\,s_{\alpha 2}\right)/$$
$$\left(\omega^{2\,\alpha_1}\,\left(1 + \tau^2\,\omega^{2\,\alpha_2} + 2\,\tau\,\omega^{\alpha_2}\,\alpha 2\right)\,{Q_1}^2 + 2\,\omega^{\alpha_1+\alpha_2}\,\left(\tau\,\omega^{\alpha_2}\,c_{\alpha 1} + c_{\alpha 1 m \alpha 2}\right)\,Q_1\,Q_2 + \omega^{2\,\alpha_2}\,{Q_2}^2\right)$$

Chapter 4

Circuits made of two Rs and two CPEs

4.1 Circuit $((R_1/Q_1)+(R_2/Q_2))$

Fig. 4.1.

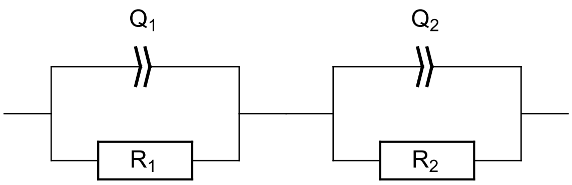

Figure 4.1: Circuit $((R_1/Q_1)+(R_2/Q_2))$.

$$Z(\omega) = \frac{1}{(\mathrm{i}\,\omega)^{\alpha_1}\,Q_1 + \dfrac{1}{R_1}} + \frac{1}{(\mathrm{i}\,\omega)^{\alpha_2}\,Q_2 + \dfrac{1}{R_2}}$$

$$Z(\omega) = \frac{R_1}{1 + (\mathrm{i}\,\omega)^{\alpha_1}\,\tau_1} + \frac{R_2}{1 + (\mathrm{i}\,\omega)^{\alpha_2}\,\tau_2}\ ,\ \tau_1 = R_1\,Q_1\ ,\ \tau_2 = R_2\,Q_2$$

$$Z(\omega) = \frac{R_1 + R_2 + (\mathrm{i}\,\omega)^{\alpha_1}\,R_2\,\tau_1 + (\mathrm{i}\,\omega)^{\alpha_2}\,R_1\,\tau_2}{(1 + (\mathrm{i}\,\omega)^{\alpha_1}\,\tau_1)\ (1 + (\mathrm{i}\,\omega)^{\alpha_2}\,\tau_2)}$$

$$\mathrm{Re}\ Z(\omega) = \frac{R_1\ (1 + \omega^{\alpha_1}\,c_{\alpha 1}\,\tau_1)}{1 + \omega^{\alpha_1}\,\tau_1\ (2\,c_{\alpha 1} + \omega^{\alpha_1}\,\tau_1)} + \frac{R_2\ (1 + \omega^{\alpha_2}\,c_{\alpha 2}\,\tau_2)}{1 + \omega^{\alpha_2}\,\tau_2\ (2\,c_{\alpha 2} + \omega^{\alpha_2}\,\tau_2)}$$

$$\mathrm{Im}\ Z(\omega) = -\frac{\omega^{\alpha_1}\,R_1\,s_{\alpha 1}\,\tau_1}{1 + \omega^{\alpha_1}\,\tau_1\ (2\,c_{\alpha 1} + \omega^{\alpha_1}\,\tau_1)} - \frac{\omega^{\alpha_2}\,R_2\,s_{\alpha 2}\,\tau_2}{1 + \omega^{\alpha_2}\,\tau_2\ (2\,c_{\alpha 2} + \omega^{\alpha_2}\,\tau_2)}$$

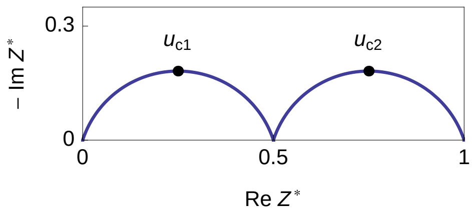

Figure 4.2: Nyquist diagrams of the reduced impedance for the ((R$_1$/Q$_1$)+(R$_2$/Q$_2$)) circuit (Fig. 4.1). $R_1 = R_2$, $\alpha_1 = \alpha_2$, $Q_2 \gg Q_1$.

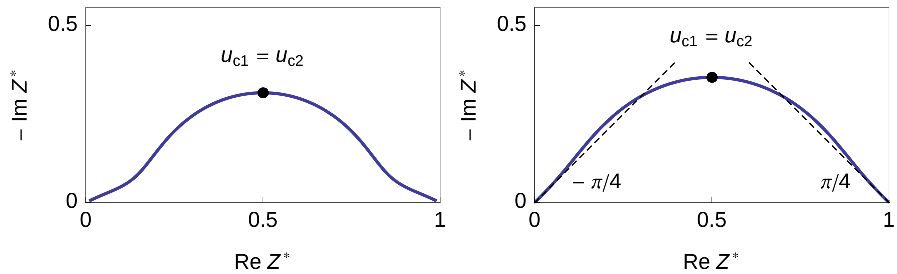

Figure 4.3: Unusual Nyquist diagrams of the reduced impedance for the ((R$_1$/Q$_1$)+(R$_2$/Q$_2$)) circuit (Fig. 4.1). $R_1 = R_2$, $Q_2 = Q_1$, $\alpha_1 = 1$. Left: $\alpha_2 = 0.3$, right: $\alpha_2 = 0.5$.

4.2 Circuit ((R$_1$+(R$_2$/Q$_2$))/Q$_1$)

Fig. 4.4.

$$Z(\omega) = \cfrac{1}{(\mathrm{i}\,\omega)^{\alpha_1} Q_1 + \cfrac{1}{R_1 + \cfrac{1}{(\mathrm{i}\,\omega)^{\alpha_2} Q_2 + \cfrac{1}{R_2}}}}$$

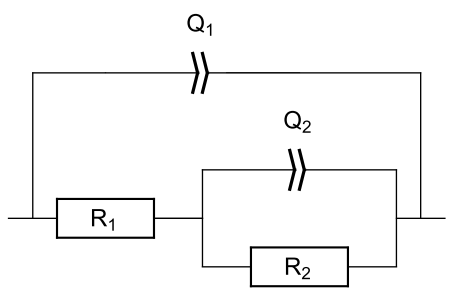

Figure 4.4: Circuit ((R$_1$+(R$_2$/Q$_2$))/Q$_1$).

$$Z(\omega) = \frac{R_1 + R_2 + (\mathrm{i}\,\omega)^{\alpha_2}\, Q_2\, R_1\, R_2}{1 + (\mathrm{i}\,\omega)^{\alpha_1}\, Q_1\, (R_1 + R_2) + (\mathrm{i}\,\omega)^{\alpha_2}\, Q_2\, R_2 + (\mathrm{i}\,\omega)^{\alpha_1+\alpha_2}\, Q_1\, Q_2\, R_1\, R_2}$$

$$\begin{gathered}
\operatorname{Re} Z(\omega) = \big(R_1 + R_2 + \omega^{2\,\alpha_2}\, {Q_2}^2\, R_1\, \left(1 + \omega^{\alpha_1}\, C_{\alpha 1}\, Q_1\, R_1\right)\, {R_2}^2 + \\
\omega^{\alpha_1}\, C_{\alpha 1}\, Q_1\, (R_1 + R_2)^2 + \omega^{\alpha_2}\, C_{\alpha 2}\, Q_2\, R_2\, \left(R_2 + 2\, R_1\, \left(1 + \omega^{\alpha_1}\, C_{\alpha 1}\, Q_1\, \left(R_1 + R_2\right)\right)\right)\big) \,/ \\
\big(1 + \omega^{2\,\alpha_2}\, {Q_2}^2\, \left(1 + \omega^{\alpha_1}\, Q_1\, R_1\, \left(2\, C_{\alpha 1} + \omega^{\alpha_1}\, Q_1\, R_1\right)\right)\, {R_2}^2 + \\
\omega^{\alpha_1}\, Q_1\, \left(R_1 + R_2\right)\, \left(2\, C_{\alpha 1} + \omega^{\alpha_1}\, Q_1\, \left(R_1 + R_2\right)\right) + 2\, \omega^{\alpha_2}\, Q_2\, R_2 \\
\times \left(C_{\alpha 2} + \omega^{\alpha_1}\, Q_1\, \left(C_{\alpha 1 m \alpha 2}\, R_2 + C_{\alpha 2}\, R_1\, \left(2\, C_{\alpha 1} + \omega^{\alpha_1}\, Q_1\, \left(R_1 + R_2\right)\right)\right)\right)\big)
\end{gathered}$$

$$c_{\alpha 1 m \alpha 2} = \cos\left(\frac{\pi\, \left(\alpha_1 - \alpha_2\right)}{2}\right)$$

$$\begin{gathered}
\operatorname{Im} Z(\omega) = \big(\omega^{\alpha_1}\, Q_1\, \big(-\omega^{2\,\alpha_2}\, {Q_2}^2\, {R_1}^2\, {R_2}^2 - 2\, \omega^{\alpha_2}\, C_{\alpha 2}\, Q_2\, R_1\, R_2\, \left(R_1 + R_2\right) - \\
\left(R_1 + R_2\right)^2\big)\, S_{\alpha 1} - \omega^{\alpha_2}\, Q_2\, {R_2}^2\, S_{\alpha 2}\big) \,/ \\
\big(1 + \omega^{2\,\alpha_2}\, {Q_2}^2\, \left(1 + \omega^{\alpha_1}\, Q_1\, R_1\, \left(2\, C_{\alpha 1} + \omega^{\alpha_1}\, Q_1\, R_1\right)\right)\, {R_2}^2 + \\
\omega^{\alpha_1}\, Q_1\, \left(R_1 + R_2\right)\, \left(2\, C_{\alpha 1} + \omega^{\alpha_1}\, Q_1\, \left(R_1 + R_2\right)\right) + 2\, \omega^{\alpha_2}\, Q_2\, R_2 \\
\times \left(C_{\alpha 2} + \omega^{\alpha_1}\, Q_1\, \left(C_{\alpha 1 m \alpha 2}\, R_2 + C_{\alpha 2}\, R_1\, \left(2\, C_{\alpha 1} + \omega^{\alpha_1}\, Q_1\, \left(R_1 + R_2\right)\right)\right)\right)\big)
\end{gathered}$$

4.3 Circuit $((\mathrm{Q}_1+(\mathrm{R}_2/\mathrm{Q}_2))/\mathrm{R}_1)$

Fig. 4.5.

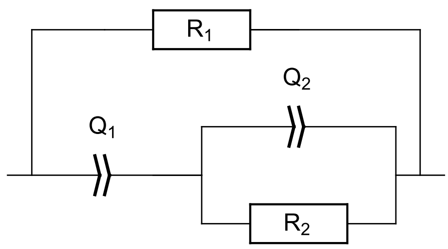

Figure 4.5: Circuit $((\mathrm{Q}_1+(\mathrm{R}_2/\mathrm{Q}_2))/\mathrm{R}_1)$.

$$Z(\omega) = \cfrac{1}{\cfrac{1}{R_1} + \cfrac{1}{\cfrac{1}{(\mathrm{i}\,\omega)^{\alpha_1}\, Q_1} + \cfrac{1}{(\mathrm{i}\,\omega)^{\alpha_2}\, Q_2 + \cfrac{1}{R_2}}}}$$

$$Z(\omega) = \frac{R_1\, \left(1 + (\mathrm{i}\,\omega)^{\alpha_1}\, Q_1\, R_2 + (\mathrm{i}\,\omega)^{\alpha_2}\, Q_2\, R_2\right)}{1 + (\mathrm{i}\,\omega)^{\alpha_1}\, Q_1\, \left(R_1 + R_2\right) + (\mathrm{i}\,\omega)^{\alpha_2}\, Q_2\, R_2 + (\mathrm{i}\,\omega)^{\alpha_1+\alpha_2}\, Q_1\, Q_2\, R_1\, R_2}$$

$$\mathrm{Re}\ Z(\omega) = \big(R_1\ \big(1+\omega^{\alpha_2}\,Q_2\,R_2\ (2\,C_{\alpha 2}+\omega^{\alpha_2}\,Q_2\,R_2)+\omega^{2\,\alpha_1}\,{Q_1}^2\,R_2 \\ \times (R_2+R_1\ (1+\omega^{\alpha_2}\,C_{\alpha 2}\,Q_2\,R_2))+\omega^{\alpha_1}\,Q_1\ (2\,R_2\ (C_{\alpha 1}+\omega^{\alpha_2}\,C_{\alpha 1 m\alpha 2}\,Q_2\,R_2)+ \\ C_{\alpha 1}\,R_1\ (1+\omega^{\alpha_2}\,Q_2\,R_2\ (2\,C_{\alpha 2}+\omega^{\alpha_2}\,Q_2\,R_2)))))\,/ \\ \big(1+\omega^{2\,\alpha_2}\,{Q_2}^2\ (1+\omega^{\alpha_1}\,Q_1\,R_1\ (2\,C_{\alpha 1}+\omega^{\alpha_1}\,Q_1\,R_1))\ {R_2}^2+ \\ \omega^{\alpha_1}\,Q_1\ (R_1+R_2)\ (2\,C_{\alpha 1}+\omega^{\alpha_1}\,Q_1\ (R_1+R_2))+ \\ 2\,\omega^{\alpha_2}\,Q_2\,R_2\ (C_{\alpha 2}+\omega^{\alpha_1}\,Q_1\ (C_{\alpha 1 m\alpha 2}\,R_2+C_{\alpha 2}\,R_1\ (2\,C_{\alpha 1}+\omega^{\alpha_1}\,Q_1\ (R_1+R_2)))))$$

$$\mathrm{Im}\ Z(\omega) = -\omega^{\alpha_1}\,Q_1\,{R_2}^2\ (S_{\alpha 1}+\omega^{\alpha_2}\,Q_2\,R_2\ ((2\,C_{\alpha 2}+\omega^{\alpha_2}\,Q_2\,R_2)\ S_{\alpha 1}+\omega^{\alpha_1}\,Q_1\,R_2\,S_{\alpha 2}))\,/ \\ \big(1+\omega^{2\,\alpha_2}\,{Q_2}^2\ (1+\omega^{\alpha_1}\,Q_1\,R_1\ (2\,C_{\alpha 1}+\omega^{\alpha_1}\,Q_1\,R_1))\ {R_2}^2+ \\ \omega^{\alpha_1}\,Q_1\ (R_1+R_2)\ (2\,C_{\alpha 1}+\omega^{\alpha_1}\,Q_1\ (R_1+R_2))+ \\ 2\,\omega^{\alpha_2}\,Q_2\,R_2\ (C_{\alpha 2}+\omega^{\alpha_1}\,Q_1\ (C_{\alpha 1 m\alpha 2}\,R_2+C_{\alpha 2}\,R_1\ (2\,C_{\alpha 1}+\omega^{\alpha_1}\,Q_1\ (R_1+R_2)))))$$

$$Z(\omega) = \frac{R_1\ (1+\tau_1\,(\mathrm{i}\,\omega)^{\alpha_1}+\tau_2\,(\mathrm{i}\,\omega)^{\alpha_2})}{1+(1+R_1/R_2)\ \tau_1\,(\mathrm{i}\,\omega)^{\alpha_1}+\tau_2\,(\mathrm{i}\,\omega)^{\alpha_2}+\tau_1\,\tau_2\,(R_1/R_2)\,(\mathrm{i}\,\omega)^{\alpha_1+\alpha_2}}$$

$$\tau_1 = Q_1\,R_2\ ,\ \tau_2 = Q_2\,R_2$$

4.4 Circuit $(((\mathrm{Q}_2+\mathrm{R}_2)/\mathrm{R}_1)/\mathrm{Q}_1)$

Fig. 4.6.

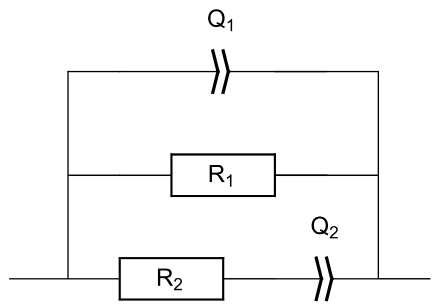

Figure 4.6: Circuit $(((\mathrm{Q}_2+\mathrm{R}_2)/\mathrm{R}_1)/\mathrm{Q}_1)$.

$$Z(\omega) = \frac{1}{(\mathrm{i}\,\omega)^{\alpha_1}\,Q_1+\dfrac{1}{R_1}+\dfrac{1}{\dfrac{1}{(\mathrm{i}\,\omega)^{\alpha_2}\,Q_2}+R_2}}$$

$$Z(\omega) = \frac{R_1\ (1+(\mathrm{i}\,\omega)^{\alpha_2}\,Q_2\,R_2)}{1+(\mathrm{i}\,\omega)^{\alpha_1}\,Q_1\,R_1+(\mathrm{i}\,\omega)^{\alpha_2}\,Q_2\,R_1+(\mathrm{i}\,\omega)^{\alpha_2}\,Q_2\,R_2+(\mathrm{i}\,\omega)^{\alpha_1+\alpha_2}\,Q_1\,Q_2\,R_1\,R_2}$$

$$\begin{aligned}\mathrm{Re}\,Z(\omega) = (&R_1\ (1+\omega^{\alpha_2}\,Q_2\ (\omega^{\alpha_2}\,Q_2\,R_2\ (R_1+R_2)+C_{\alpha 2}\ (R_1+2\,R_2))+\\
&\omega^{\alpha_1}\,C_{\alpha 1}\,Q_1\,R_1\ (1+\omega^{\alpha_2}\,Q_2\,R_2\ (2\,C_{\alpha 2}+\omega^{\alpha_2}\,Q_2\,R_2))))\,/\\
&(1+\omega^{\alpha_2}\,Q_2\ (R_1+R_2)\ (2\,C_{\alpha 2}+\omega^{\alpha_2}\,Q_2\ (R_1+R_2))+\\
&\omega^{2\,\alpha_1}\,{Q_1}^2\,{R_1}^2\ (1+\omega^{\alpha_2}\,Q_2\,R_2\ (2\,C_{\alpha 2}+\omega^{\alpha_2}\,Q_2\,R_2))+2\,\omega^{\alpha_1}\,Q_1\,R_1\\
&\times(C_{\alpha 1}+\omega^{\alpha_2}\,Q_2\ (C_{\alpha 1 m\alpha 2}\,R_1+2\,C_{\alpha 1}\,C_{\alpha 2}\,R_2+\omega^{\alpha_2}\,C_{\alpha 1}\,Q_2\,R_2\ (R_1+R_2))))\end{aligned}$$

$$\begin{aligned}\mathrm{Im}\,Z(\omega) = \big(&{R_1}^2\ (-(\omega^{\alpha_1}\,Q_1\ (1+\omega^{\alpha_2}\,Q_2\,R_2\ (2\,C_{\alpha 2}+\omega^{\alpha_2}\,Q_2\,R_2))\ S_{\alpha 1})-\omega^{\alpha_2}\,Q_2\,S_{\alpha 2})\big)\,/\\
&(1+\omega^{\alpha_2}\,Q_2\ (R_1+R_2)\ (2\,C_{\alpha 2}+\omega^{\alpha_2}\,Q_2\ (R_1+R_2))+\\
&\omega^{2\,\alpha_1}\,{Q_1}^2\,{R_1}^2\ (1+\omega^{\alpha_2}\,Q_2\,R_2\ (2\,C_{\alpha 2}+\omega^{\alpha_2}\,Q_2\,R_2))+2\,\omega^{\alpha_1}\,Q_1\,R_1\\
&\times(C_{\alpha 1}+\omega^{\alpha_2}\,Q_2\ (C_{\alpha 1 m\alpha 2}\,R_1+2\,C_{\alpha 1}\,C_{\alpha 2}\,R_2+\omega^{\alpha_2}\,C_{\alpha 1}\,Q_2\,R_2\ (R_1+R_2))))\end{aligned}$$

$$Z(\omega) = \frac{R_1\ (1+(\mathrm{i}\,\omega)^{\alpha_2}\,\tau_2)}{1+(\mathrm{i}\,\omega)^{\alpha_1}\,\tau_1+(1+R_1/R_2)\ (\mathrm{i}\,\omega)^{\alpha_2}\,\tau_2+(\mathrm{i}\,\omega)^{\alpha_1+\alpha_2}\,\tau_1\,\tau_2}$$
$$\tau_1 = Q_1\,R_1\ ,\ \tau_2 = Q_2\,R_2$$

Appendix A

Symbols for CPE

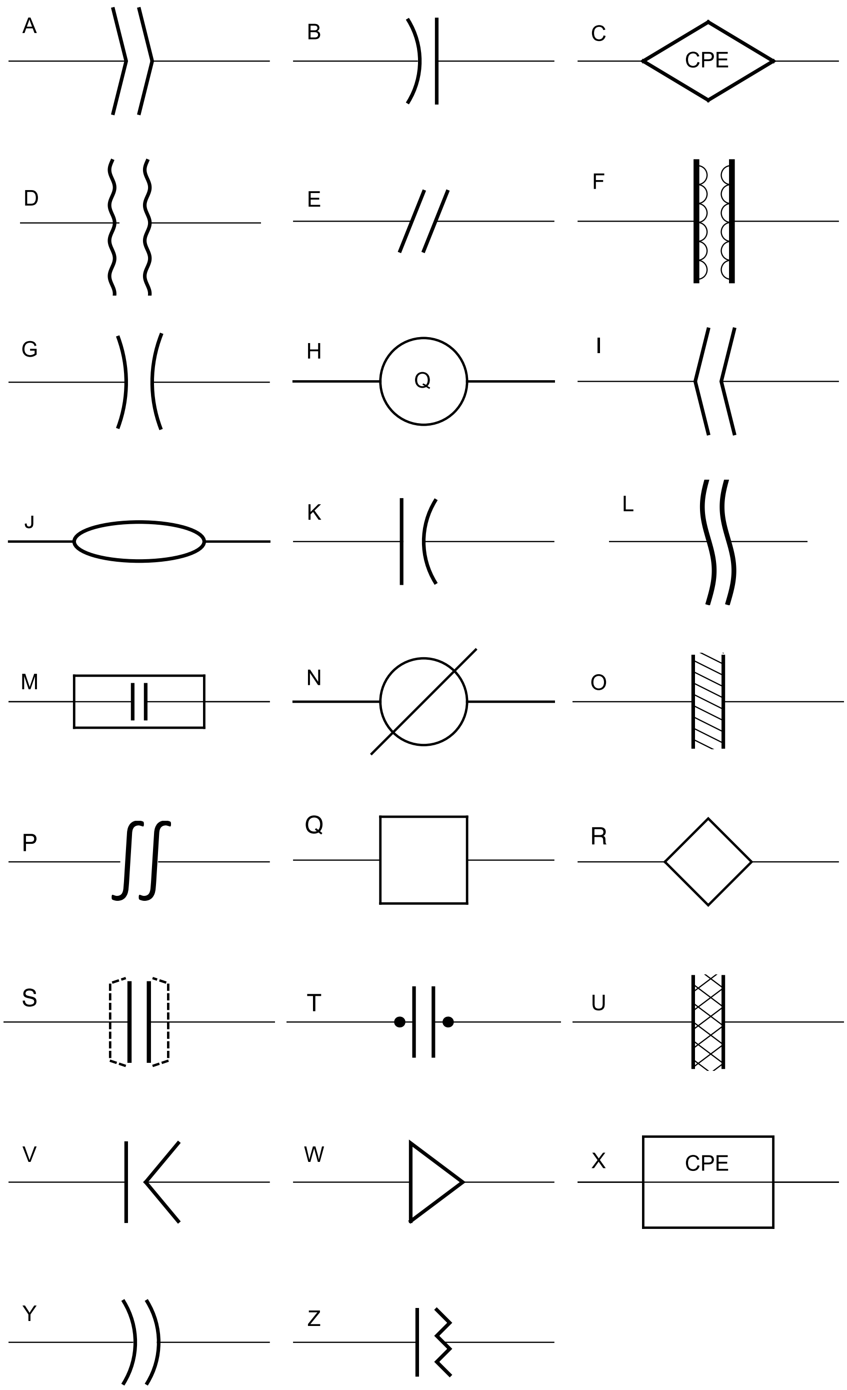

Figure A.1: Some CPE symbols, taken from A: [16], B: [24], C: [29], D: [6], E: [13], F: [21], G: [22], H: [27], I: [15], J: [19], K: [25], L: [3, 11], M: [14], N: [18, 4, 5], O: [17], P: [30], Q, R: [31], S: [9], T: [20], U [26], V: [12], W: [32], X: [8], Y: [2], Z: [23]. What an imagination !

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