Handbook of Electrochemical Impedance Spectroscopy CIRCUITS made of RESISTORS and CAPACITORS

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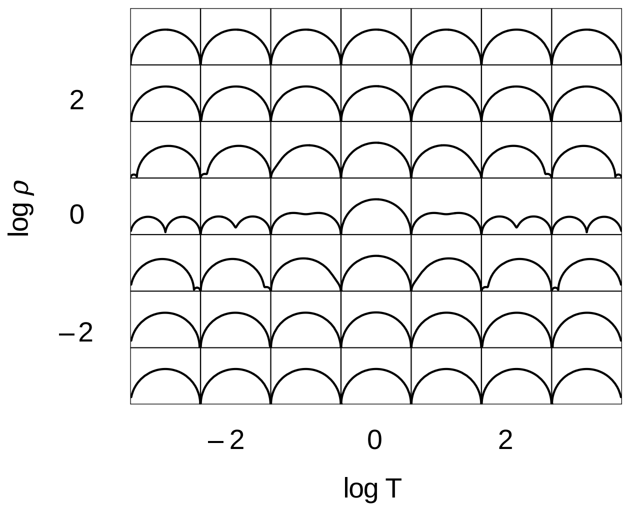
Electrochemical Impedance Spectroscopy



CIRCUITS
made of
RESISTORS and CAPACITORS

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Chapter 1

Circuits made of one R and one C

1.1 Circuit (R+C)

The symbol '+' denotes the serial association of electrical components.

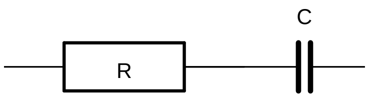

Figure 1.1: Circuit (R+C).

1.1.1 Impedance

$$Z(\omega) = R + \frac{1}{C\,\mathrm{i}\,\omega} = \frac{R\,(1 + \mathrm{i}\,\omega\,\tau)}{\mathrm{i}\,\omega\,\tau}\ ,\ \tau = R\,C$$

$$\mathrm{Re}\ Z(\omega) = R\ ,\ \mathrm{Im}\ Z(\omega) = -\frac{R}{\tau\,\omega}$$

1.1.2 Reduced impedance

$$Z^*(u) = Z/R = 1 + \frac{1}{\mathrm{i}\,u} = \frac{1 + \mathrm{i}\,u}{\mathrm{i}\,u}\ ,\ u = \tau\,\omega\ ,\ \mathrm{Re}\ Z^*(u) = 1\ ,\ \mathrm{Im}\ Z^*(u) = -\frac{1}{u} \tag{1.1}$$

(Fig. 1.2)

1.2 Circuit (R/C)

The symbol '/' denotes the parallel association of electrical components.

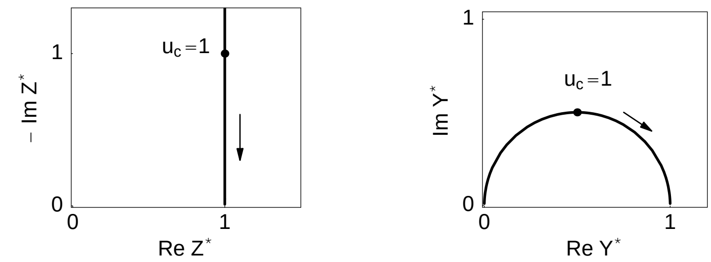

Figure 1.2: Nyquist diagram of the reduced impedance and admittance ($Y^* = RY$) for the (R+C) circuit (Fig. 1.1, Eq. (1.1)). The arrows always indicate the increasing frequency direction.

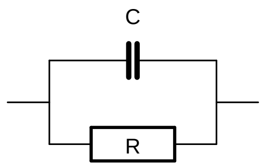

Figure 1.3: Circuit (R/C).

1.2.1 Impedance

$$Z(\omega) = \frac{R}{1 + \mathrm{i}\,\omega\,\tau}\ ,\ \tau = R\,C$$

$$\text{Re } Z(\omega) = \frac{R}{1 + \tau^2\,\omega^2},\ \text{Im } Z(\omega) = -\frac{R\,\tau\,\omega}{1 + \tau^2\,\omega^2}$$

1.2.2 Reduced impedance

$$Z^*(u) = \frac{Z(u)}{R} = \frac{1}{1 + \mathrm{i}\,u},\quad u = \tau\,\omega,\ \text{Re } Z^*(u) = \frac{1}{1 + u^2},\ \text{Im } Z^*(u) = -\frac{u}{1 + u^2} \tag{1.2}$$

(Fig. 1.4)

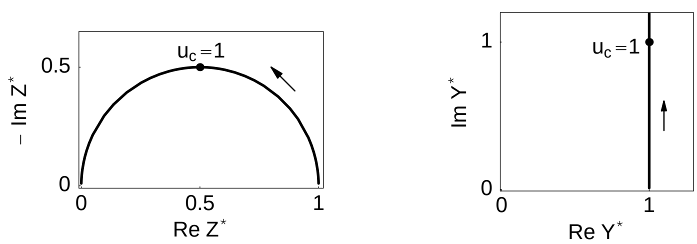

Figure 1.4: Nyquist diagram of the reduced impedance and admittance ($Y^* = RY$) for the (R/C) circuit (Fig. 1.3, Eq. (1.2)).

Chapter 2

Circuits made of two Rs and one C

2.1 Circuit (R_2+(R_1/C_1))

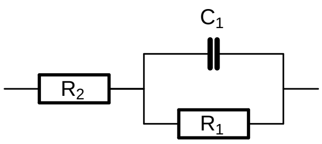

Figure 2.1: Circuit (R_2+(R_1/C_1)).

2.1.1 Impedance

$$Z(\omega) = R_2 + \frac{1}{\mathrm{i}\,\omega\,C_1 + \dfrac{1}{R_1}}$$

$$Z(\omega) = \frac{(R_1 + R_2)\ (1 + \mathrm{i}\,\omega\,\tau_2)}{1 + \mathrm{i}\,\omega\,\tau_1}\ ,\ \tau_1 = R_1\,C_1\ ,\ \tau_2 = \frac{C_1\,R_1\,R_2}{R_1 + R_2}$$

2.1.2 Reduced impedance

$$Z^*(u) = \frac{Z(u)}{R_1 + R_2} = \frac{1 + T\,\mathrm{i}\,u}{1 + \mathrm{i}\,u} \tag{2.1}$$

$$u = \tau_1\,\omega\ ,\ T = \tau_2/\tau_1 = R_2/\,(R_1 + R_2) < 1$$

$$\mathrm{Re}\ Z^*(u) = \frac{1 + T\,u^2}{1 + u^2}\ ,\ \mathrm{Im}\ Z^*(u) = \frac{(T-1)\ u}{1 + u^2}$$

$$\lim_{u\to 0} \mathrm{Re}\ Z^*(u) = 1,\ \lim_{u\to\infty} \mathrm{Re}\ Z^*(u) = T$$

(Fig. 2.2)

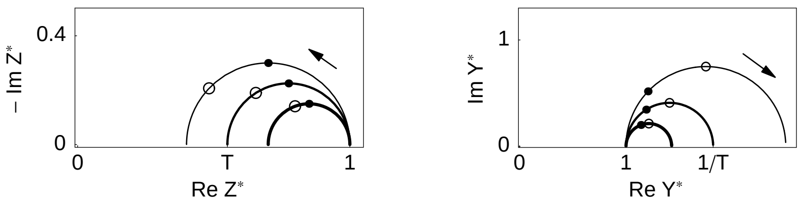

Figure 2.2: Nyquist diagram of the reduced impedance and admittance ($Y^* = (R_1 + R_2)Y$) for the (R_2+(R_1/C_1)) circuit (Fig. 2.1, Eq. (2.1)) with $T = 0.4, 0.55, 0.7$. The line thickness increases with increasing T). Dots: reduced characteristic angular frequency $u_{c1} = 1$; circles: reduced characteristic angular frequency $u_{c2} = 1/T$.

2.2 Circuit ((R_1+C_1)/R_2)

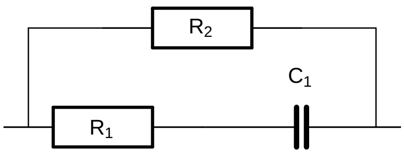

Figure 2.3: Circuit ((R_1+C_1)/R_2).

2.2.1 Impedance

$$Z(\omega) = \frac{R_2\ (1 + \mathrm{i}\,\omega\,\tau_2)}{1 + \mathrm{i}\,\omega\,\tau_1}\ ,\ \tau_1 = C_1\ (R_1 + R_2)\ ,\ \tau_2 = R_1\,C_1$$

$$\mathrm{Re}\ Z(\omega) = \frac{R_2\ \left(1 + \omega^2\,\tau_1\,\tau_2\right)}{1 + \omega^2\,{\tau_1}^2}\ ,\ \mathrm{Im}\ Z(\omega) = \frac{\omega\,R_2\ (-\tau_1 + \tau_2)}{1 + \omega^2\,{\tau_1}^2}$$

2.2.2 Reduced impedance

$$Z^*(u) = Z(u)/R_2 = \frac{1 + T\,\mathrm{i}\,u}{1 + \mathrm{i}\,u} \tag{2.2}$$

$$u = \tau_1\,\omega\ ,\ T = \tau_2/\tau_1 = R_1/(R_1 + R_2) < 1$$

cf. Eq. (2.1) and Fig. 2.2.

2.3 Transformation formulae between (R+(R/C)) and ((R+C)/R) circuits

2.3.1 Transformation formulae (R+(R/C)) → ((R+C)/R)

$$R_{22} = R_{11} + R_{21}, R_{12} = R_{11} + \frac{{R_{11}}^2}{R_{21}}, C_{12} = \frac{C_{11}\,{R_{21}}^2}{\left(R_{11} + R_{21}\right)^2}$$

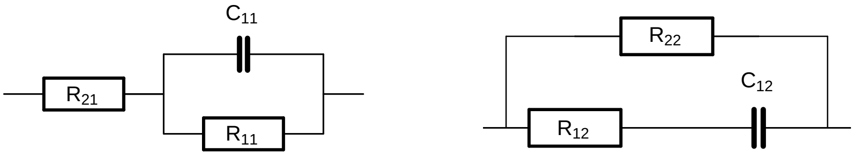

Figure 2.4: The (R+(R/C)) and ((R+C)/R circuits are non-distinguishable [3, 2, 1, 6].

2.3.2 Transformation formulae ((R+C)/R) → (R+(R/C))

$$C_{11} = \frac{C_{12}\,(R_{12}+R_{22})^2}{{R_{22}}^2}, R_{11} = \frac{R_{12}\,R_{22}}{R_{12}+R_{22}}, R_{21} = \frac{{R_{22}}^2}{R_{12}+R_{22}}$$

Chapter 3

Circuits made of one R and two Cs

3.1 Circuit ((R$_1$/C$_1$)+C$_2$)

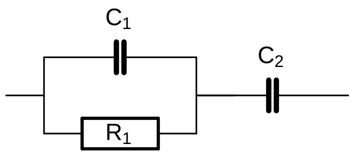

Figure 3.1: Circuit ((R$_1$/C$_1$)+C$_2$).

3.1.1 Impedance

$$Z(\omega) = \frac{1}{\mathrm{i}\,\omega\,C_1 + \dfrac{1}{R_1}} + \frac{1}{\mathrm{i}\,\omega\,C_2} = \frac{1 + \mathrm{i}\,\omega\;(C_1 + C_2)\;R_1}{\mathrm{i}\,\omega\,C_2\;(1 + \mathrm{i}\,\omega\,C_1\,R_1)}$$

$$Z(\omega) = \frac{1 + \mathrm{i}\,\omega\,\tau_2}{\mathrm{i}\,\omega\,C_2\;(1 + \mathrm{i}\,\omega\,\tau_1)}\;,\;\tau_1 = R_1\,C_1\;,\;\tau_2 = (C_1 + C_2)\,R_1\;,\;\tau_1 < \tau_2$$

$$\mathrm{Re}\;Z(\omega) = -\frac{\tau_1 - \tau_2}{C_2\;(1 + \omega^2\,\tau_1{}^2)}\;,\;\mathrm{Im}\;Z(\omega) = -\frac{1 + \omega^2\,\tau_1\,\tau_2}{\omega\,C_2\;(1 + \omega^2\,\tau_1{}^2)}$$

$$\lim_{\omega\to 0}\mathrm{Re}\;Z(\omega) = R_1$$

3.1.2 Reduced impedance

$$Z^*(u) = \frac{Z(u)}{R_1} = \frac{1}{T-1}\,\frac{1 + T\,\mathrm{i}\,u}{\mathrm{i}\,u\;(1 + \mathrm{i}\,u)} \tag{3.1}$$

$$u = \omega\,\tau_1\;,\;T = \tau_2/\tau_1 = 1 + C_2/C_1 > 1$$

$$\text{Re } Z^*(u) = \frac{1}{1+u^2} \; , \; \text{Im } Z^*(u) = -\frac{1+u^2\,T}{(T-1)\,u\,(1+u^2)}$$

$$\lim_{u\to 0} \text{Re } Z^*(u) = 1$$

(Fig. 3.2)

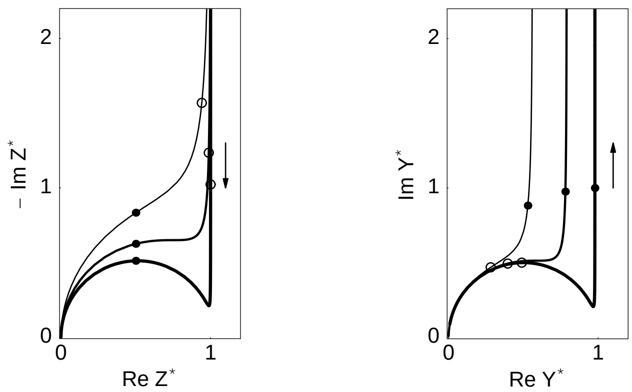

Figure 3.2: Nyquist diagram of the reduced impedance and admittance ($Y^* = R_1\,Y$) for the ($(R_1/C_1)+C_2$) circuit (Fig. 3.1, Eq. (3.1)) plotted for $T = 4, 9, 90$. The line thickness increases with increasing T. Horizontal tangent is observed for $T \geq 9$ ($C_2/C_1 \geq 8$) [4]. Dots: reduced characteristic angular frequency $u_{c1} = 1$; circles: reduced characteristic angular frequency $u_{c2} = 1/T$.

3.2 Circuit ((R$_1$+C$_2$)/C$_1$)

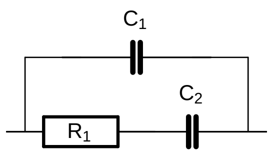

Figure 3.3: Circuit ($(R_1+C_2)/C_1$).

3.2.1 Impedance

$$Z(\omega) = \frac{1}{\mathrm{i}\,\omega\,C_1 + \dfrac{1}{R_1 + \dfrac{1}{\mathrm{i}\,\omega\,C_2}}} = \frac{1 + \mathrm{i}\,\omega\,C_2\,R_1}{\mathrm{i}\,\omega\,(C_1+C_2)\left(1 + \dfrac{\mathrm{i}\,\omega\,C_1\,C_2\,R_1}{C_1+C_2}\right)}$$

$$Z(\omega) = \frac{(1+\mathrm{i}\,\omega\,\tau_2)}{\mathrm{i}\,\omega\,(C_1+C_2)\,(1+\mathrm{i}\,\omega\,\tau_1)} \; , \; \tau_1 = \frac{C_1\,C_2\,R_1}{C_1+C_2} \; , \; \tau_2 = C_2\,R_1 \qquad (3.2)$$

$$\mathrm{Re}\,Z(\omega) = \frac{-\tau_1 + \tau_2}{(C_1 + C_2)\,(1 + \omega^2\,\tau_1{}^2)}\ ,\ \mathrm{Im}\,Z(\omega) = -\frac{1 + \omega^2\,\tau_1\,\tau_2}{\omega\,(C_1 + C_2)\,(1 + \omega^2\,\tau_1{}^2)}$$

$$\lim_{\omega\to 0} \mathrm{Re}\,Z(\omega) = \frac{C_2{}^2\,R_1}{(C_1 + C_2)^2}$$

3.2.2 Reduced impedance

$$Z^*(u) = \frac{Z(u)}{R_1} = \frac{T-1}{T^2}\,\frac{1 + T\,\mathrm{i}\,u}{\mathrm{i}\,u\,(1 + \mathrm{i}\,u)} \tag{3.3}$$

$$u = \omega\,\tau_1\ ,\ T = \tau_2/\tau_1 = 1 + C_2/C_1 > 1$$

$$\mathrm{Re}\,Z^*(u) = \frac{(-1+T)^2}{T^2\,(1+u^2)}\ ,\ \mathrm{Im}\,Z^*(u) = -\frac{(-1+T)\,\left(1 + T\,u^2\right)}{T^2\,u\,(1+u^2)}$$

$$\lim_{u\to 0} \mathrm{Re}\,Z^*(u) = \frac{(-1+T)^2}{T^2} = \frac{C_2{}^2}{(C_1 + C_2)^2}$$

(Fig. 3.4)

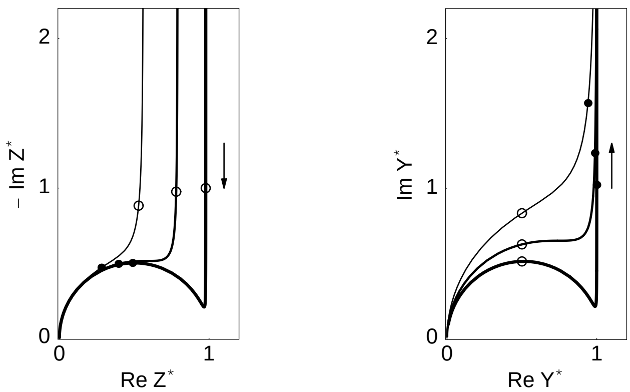

Figure 3.4: Nyquist diagram of the reduced impedance and admittance ($Y^* = R_1\,Y$) for the ((R_1+C_2)/C_1) circuit (Fig. 3.3, Eq. (3.3)) plotted for $T = 4, 9, 90$). The line thickness increases with increasing T. Horizontal tangent is observed for $T \geq 9$. ($C_2/C_1 \geq 8$). Dots: reduced characteristic angular frequency $u_{c1} = 1$; circles: reduced characteristic angular frequency: $u_{c2} = 1/T$.

3.3 Transformation formulae between ((R/C)/C) and ((R+C)/C) circuits

3.3.1 Transformation formulae ((R/C)/C) → ((R+C)/C)

$$C_{22} = \frac{C_{11}\,C_{21}}{C_{11} + C_{21}}, R_{12} = \frac{(C_{11} + C_{21})^2\,R_{11}}{C_{21}{}^2}, C_{12} = \frac{C_{21}{}^2}{C_{11} + C_{21}}$$

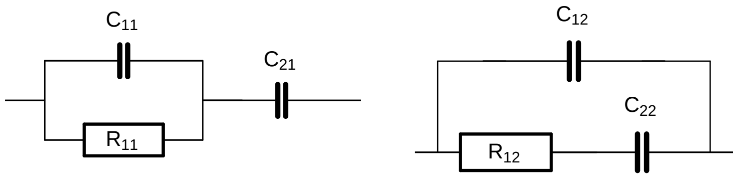

Figure 3.5: The ((R/C)/C) and ((R+C)/C) circuits are non-distinguishable [3, 2, 1, 6].

3.3.2 Transformation formulae ((R+C)/C) → ((R/C)/C)

$$C_{11} = C_{22} + \frac{{C_{22}}^2}{C_{12}}, R_{11} = \frac{{C_{12}}^2 \, R_{12}}{(C_{12} + C_{22})^2}, C_{21} = C_{12} + C_{22}$$

Chapter 4

Circuits made of two Rs and two Cs

4.1 Circuit ((R$_1$/C$_1$)+(R$_2$/C$_2$))

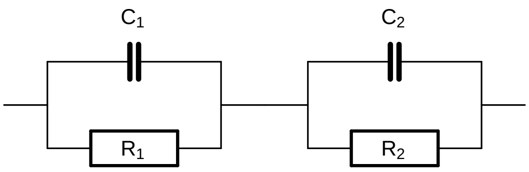

Figure 4.1: Circuit ((R$_1$/C$_1$)+(R$_2$/C$_2$)) .

4.1.1 Impedance

$$Z(\omega) = \frac{1}{\mathrm{i}\,\omega\, C_1 + \dfrac{1}{R_1}} + \frac{1}{\mathrm{i}\,\omega\, C_2 + \dfrac{1}{R_2}} = \frac{(R_1 + R_2)\;(1 + \mathrm{i}\,\omega\,\tau_3)}{(1 + \mathrm{i}\,\omega\,\tau_1)\;(1 + \mathrm{i}\,\omega\,\tau_2)}$$

$$\tau_1 = R_1\, C_1 \;,\; \tau_2 = R_2\, C_2 \;,\; \tau_3 = \frac{(C_1 + C_2)\; R_1\, R_2}{R_1 + R_2} = \frac{\tau_1\, R_2 + \tau_2\, R_1}{R_1 + R_2}$$

$$\mathrm{Re}\; Z(\omega) = \frac{(R_1 + R_2)\;\left(1 + \omega^2\;(-\tau_1\,\tau_2 + (\tau_1 + \tau_2)\;\tau_3)\right)}{(1 + \omega^2\,{\tau_1}^2)\;(1 + \omega^2\,{\tau_2}^2)}$$

$$\mathrm{Im}\; Z(\omega) = -\frac{\omega\;(R_1 + R_2)\;\left(\tau_1 + \tau_2 + \left(-1 + \omega^2\,\tau_1\,\tau_2\right)\;\tau_3\right)}{(1 + \omega^2\,{\tau_1}^2)\;(1 + \omega^2\,{\tau_2}^2)}$$

4.1.2 Reduced impedance

$$Z^*(u) = \frac{Z(u)}{R_1 + R_2} = \frac{1 + \rho + (T + \rho)\;\mathrm{i}\,u}{(1 + \rho)\,(1 + \mathrm{i}\,u)\;(1 + \mathrm{i}\,u\,T)}$$

$$u = R_2\, C_2\,\omega \;,\; \rho = R_1/R_2 \;,\; T = R_1\, C_1/\,(R_2\, C_2) = \gamma\,\rho \;,\; \gamma = C_1/C_2$$

(Figs. 4.2-4.5)

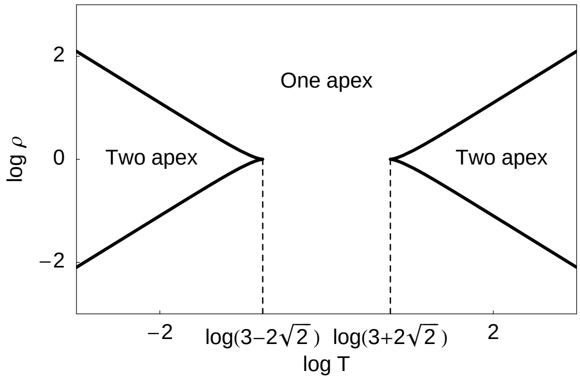

Figure 4.2: Case diagram for the ((R$_1$/C$_1$)+(R$_2$/C$_2$)) circuit plotted using the $\log \rho$ *vs.* $\log T$ representation.

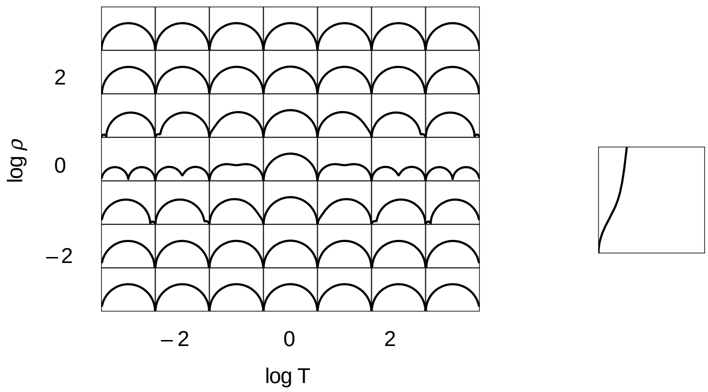

Figure 4.3: Array of impedance diagrams plotted for the ((R$_1$/C$_1$)+(R$_2$/C$_2$)) circuit, depending on ρ and T values, and enlargment of the high frequency part of the diagram calculated for $T = 10^{-2}$ and $\rho = 10^{-2}$.

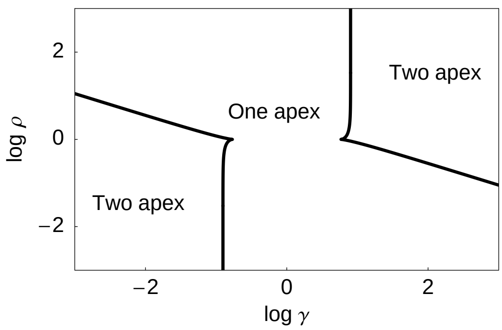

Figure 4.4: Case diagram for the ((R$_1$/C$_1$)+(R$_2$/C$_2$)) circuit, using the $\log \rho$ *vs.* $\log \gamma$ representaion.

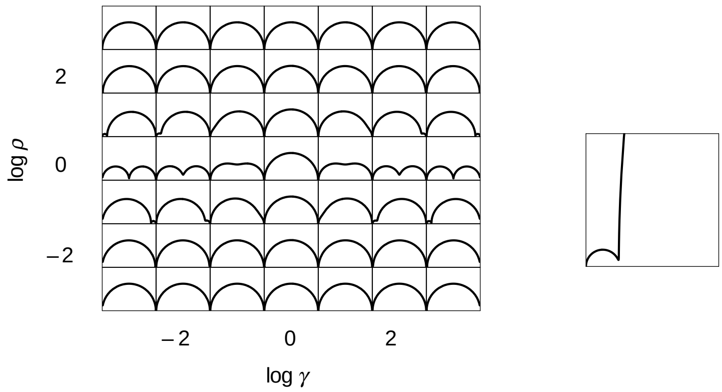

Figure 4.5: Array of impedance diagrams plotted for the $((R_1/C_1)+(R_2/C_2))$ circuit, depending on ρ and γ values and enlargment of the high frequency part of the diagram calculated for $\gamma = 10^{-2}$ and $\rho = 10^{-2}$.

4.2 Circuit ((R$_1$+(R$_2$/C$_2$))/C$_1$)

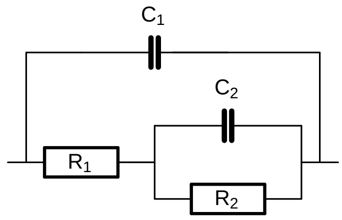

Figure 4.6: Circuit ((R$_1$+(R$_2$/C$_2$))/C$_1$).

4.2.1 Impedance

$$Z(\omega) = \frac{1}{\mathrm{i}\,\omega\,C_1 + \frac{1}{R_1 + \frac{R_2}{1+\mathrm{i}\,\omega\,C_2\,R_2}}}$$

$$Z(\omega) = \frac{(R_1+R_2)\left(1+\frac{\mathrm{i}\,\omega\,C_2\,R_1\,R_2}{R_1+R_2}\right)}{1+\mathrm{i}\,\omega\ (C_2\,R_2 + C_1\ (R_1+R_2)) + (\mathrm{i}\,\omega)^2\,C_1\,C_2\,R_1\,R_2}$$

$$Z(\omega) = \frac{(R_1+R_2)\ (1+\mathrm{i}\,\omega\,\tau_3)}{(1+\mathrm{i}\,\omega\,\tau_1)\ (1+\mathrm{i}\,\omega\,\tau_2)}\ ,\ \tau_3 = \frac{C_2\,R_1\,R_2}{R_1+R_2}$$

The poles of the impedance of a circuit made of Rs and Cs are always real [5].

$$\tau_1 = \frac{C_2\,R_2 + C_1\ (R_1+R_2) - \sqrt{-4\,C_1\,C_2\,R_1\,R_2 + (C_2\,R_2 + C_1\ (R_1+R_2))^2}}{2} \tag{4.1}$$

$$\tau_2 = \frac{C_2\,R_2 + C_1\ (R_1+R_2) + \sqrt{-4\,C_1\,C_2\,R_1\,R_2 + (C_2\,R_2 + C_1\ (R_1+R_2))^2}}{2} \tag{4.2}$$

4.2.2 Reduced impedance

$$Z^*(u) = \frac{Z(u)}{R_1+R_2} = \frac{\rho}{1+\rho}\ \frac{1+\rho\,(1+\mathrm{i}\,u)}{\mathrm{i}\,u\,T + \rho\ (1+\mathrm{i}\,u)\ (1+\mathrm{i}\,u\,T)}$$

$$u = R_2\,C_2\,\omega\ ,\ \rho = R_1/R_2\ ,\ T = R_1\,C_1/\,(R_2\,C_2) = \gamma\,\rho\ ,\ \gamma = C_1/C_2$$

(Figs. 4.7-4.10)

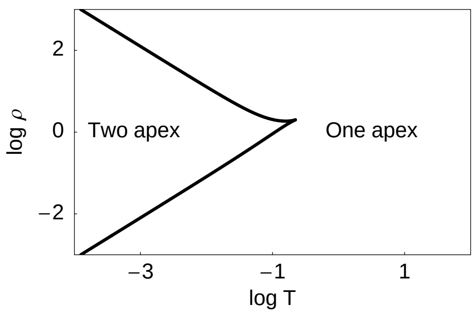

Figure 4.7: Case diagram for the $((\mathrm{R}_1+(\mathrm{R}_2/\mathrm{C}_2))/\mathrm{C}_1)$ circuit plotted using the $\log\rho$ *vs.* $\log T$ representation.

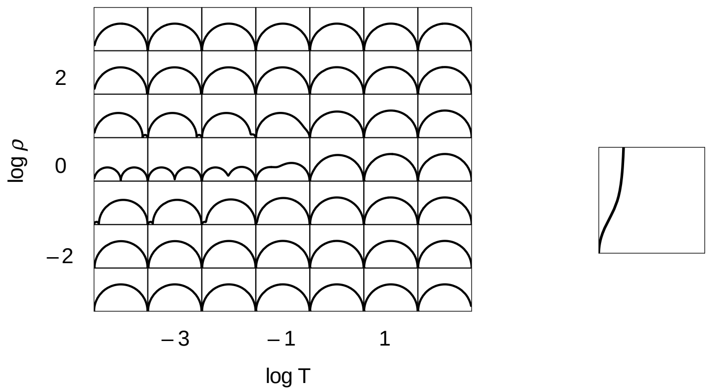

Figure 4.8: Array of impedance diagrams plotted for the $((\mathrm{R}_1+(\mathrm{R}_2/\mathrm{C}_2))/\mathrm{C}_1)$ circuit, depending on ρ and T values, and enlargment of the high frequency part of the diagram calculated for $T = 10^{-3}$ and $\rho = 10^{-3}$.

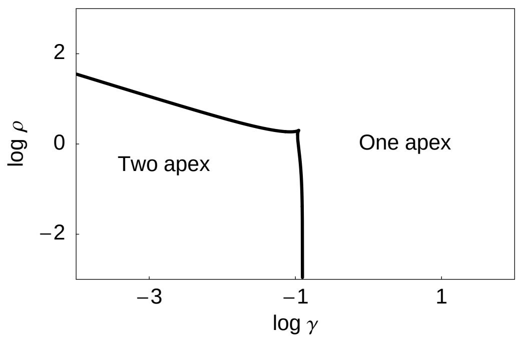

Figure 4.9: Case diagram for the $((\mathrm{R}_1+(\mathrm{R}_2/\mathrm{C}_2))/\mathrm{C}_1)$ circuit plotted using the $\log\rho$ *vs.* $\log\gamma$ representation.

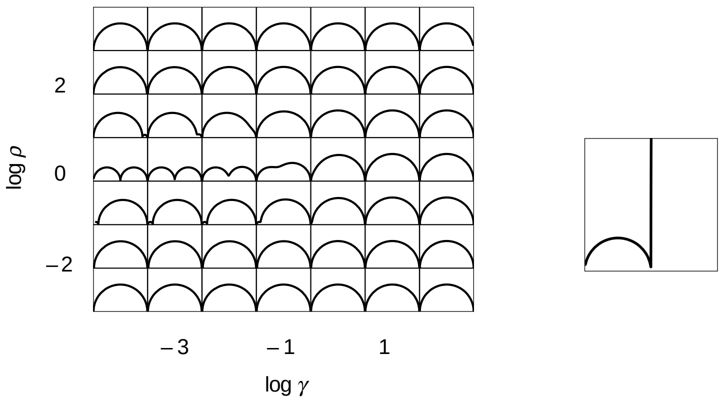

Figure 4.10: Array of impedance diagrams plotted for the $((R_1+(R_2/C_2))/C_1)$ circuit, depending on ρ and γ values, and enlargment of the high frequency part of the diagram calculated for $\gamma = 10^{-3}$ and $\rho = 10^{-3}$.

4.3 Circuit $((C_1+(R_2/C_2))/R_1)$

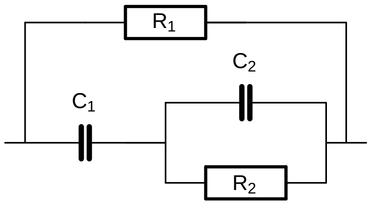

Figure 4.11: Circuit $((C_1+(R_2/C_2))/R_1)$.

4.3.1 Impedance

$$Z(\omega) = \frac{1}{\dfrac{1}{R_1} + \dfrac{1}{\dfrac{1}{\mathrm{i}\,\omega\,C_1} + \dfrac{R_2}{1+\mathrm{i}\,\omega\,C_2\,R_2}}}$$

$$Z(\omega) = \frac{R_1\,(1+\mathrm{i}\,\omega\,(C_1+C_2)\,R_2)}{1+\mathrm{i}\,\omega\,(C_2\,R_2+C_1\,(R_1+R_2))+(\mathrm{i}\,\omega)^2\,C_1\,C_2\,R_1\,R_2}$$

$$Z(\omega) = \frac{R_1\,(1+\mathrm{i}\,\omega\,\tau_3)}{(1+\mathrm{i}\,\omega\,\tau_1)\,(1+\mathrm{i}\,\omega\,\tau_2)}\ ,\ \tau_3 = (C_1+C_2)\,R_2\ ,\ \tau_1 : \text{Éq. (4.1)}\ ,\ \tau_2 : \text{Éq. (4.2)}$$

4.3.2 Reduced impedance

$$Z^*(u) = Z(u)/R_1 = \frac{\rho+\mathrm{i}\,u\,(\rho+T)}{\mathrm{i}\,u\,T+\rho\,(1+\mathrm{i}\,u)\,(1+\mathrm{i}\,u\,T)}$$

$$u = R_2\,C_2\,\omega\ ,\ \rho = R_1/R_2\ ,\ T = R_1\,C_1/\,(R_2\,C_2) = \gamma\,\rho\ ,\ \gamma = C_1/C_2$$

(Figs. 4.12 and 4.13)

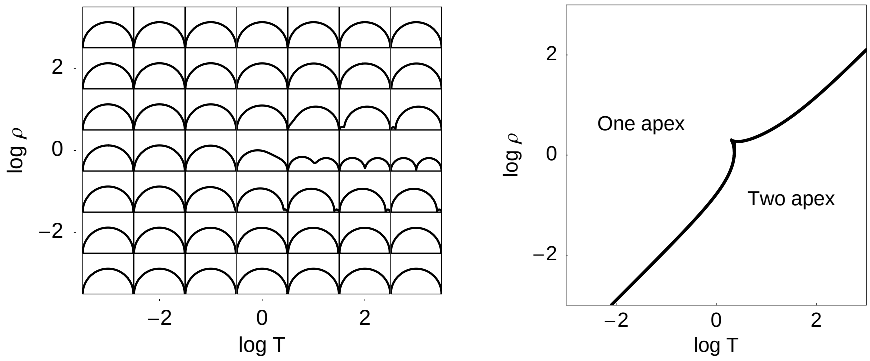

Figure 4.12: Impedance diagrams array and case diagram for the $((C_1+(R_2/C_2))/R_1)$ circuit using the $\log\rho$ *vs.* $\log T$ representation.

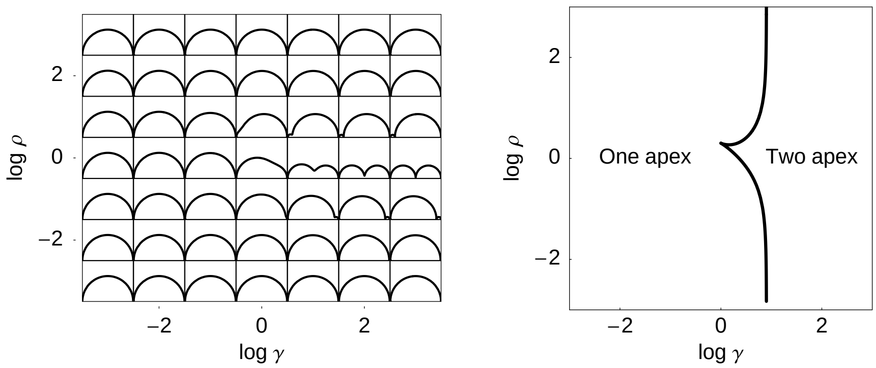

Figure 4.13: Impedance diagrams array and case diagram for the $((C_1+(R_2/C_2))/R_1)$ circuit using the $\log\rho$ *vs.* $\log\gamma$ representation.

4.4 Circuit $(((C_2{+}R_2)/R_1)/C_1)$

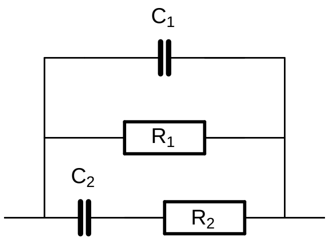

Figure 4.14: Circuit $(((C_2{+}R_2)/R_1)/C_1)$.

4.4.1 Impedance

$$Z(\omega) = \frac{1}{\mathrm{i}\,\omega\,C_1 + \dfrac{1}{R_1} + \dfrac{1}{\dfrac{1}{\mathrm{i}\,\omega\,C_2} + R_2}}$$

$$Z(\omega) = \frac{R_1\ (1 + \mathrm{i}\,\omega\,C_2\,R_2)}{1 + \mathrm{i}\,\omega\ (C_1\,R_1 + C_2\ (R_1 + R_2)) + (\mathrm{i}\,\omega)^2\,C_1\,C_2\,R_1\,R_2}$$

$$Z(\omega) = \frac{R_1\ (1 + \mathrm{i}\,\omega\,\tau_3)}{(1 + \mathrm{i}\,\omega\,\tau_1)\ (1 + \mathrm{i}\,\omega\,\tau_2)}$$

$\tau_3 = R_2\,C_2$; τ_1 : Eq. (4.1) , τ_2 : Éq. (4.2) (exchanging subscripts 1 and 2)

4.4.2 Reduced impedance

$$Z^*(u) = Z(u)/R_1 = \frac{1 + \mathrm{i}\,u}{1 + \mathrm{i}\,u\ (1 + \rho + \tau\ (1 + \mathrm{i}\,u))}$$

$$u = R_2\,C_2\,\omega\ ,\ \rho = R_1/R_2\ ,\ T = R_1\,C_1/\,(R_2\,C_2) = \gamma\,\rho\ ,\ \gamma = C_1/C_2$$

(Figs. 4.15 and 4.16)

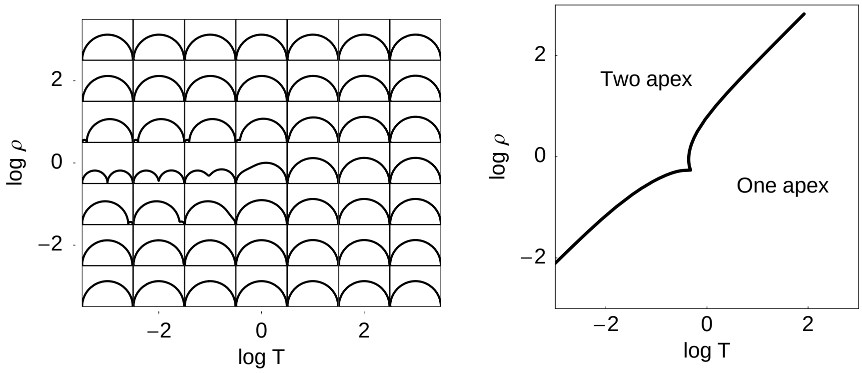

Figure 4.15: Impedance diagrams array and case diagram for the $(((C_2{+}R_2)/R_1)/C_1)$ circuit using the $\log\rho$ *vs.* $\log T$ representation.

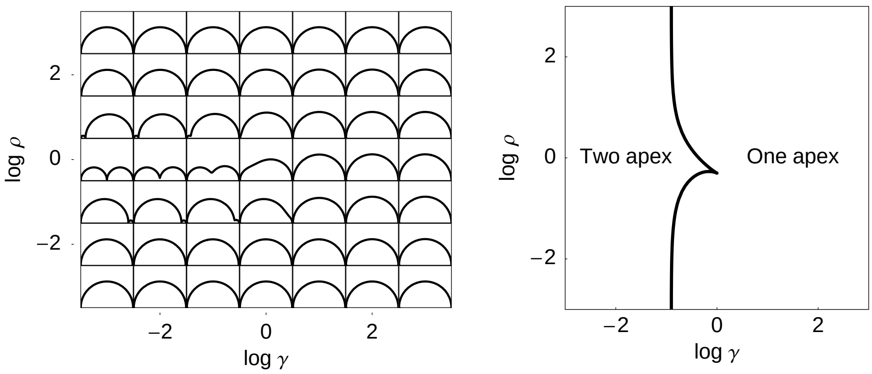

Figure 4.16: Impedance diagrams array and case diagram for the $(((C_2{+}R_2)/R_1)/C_1)$ circuit using the $\log\rho$ *vs.* $\log\gamma$ representation.

4.5 Transformation formulae for the four circuits made of two Rs and two Cs

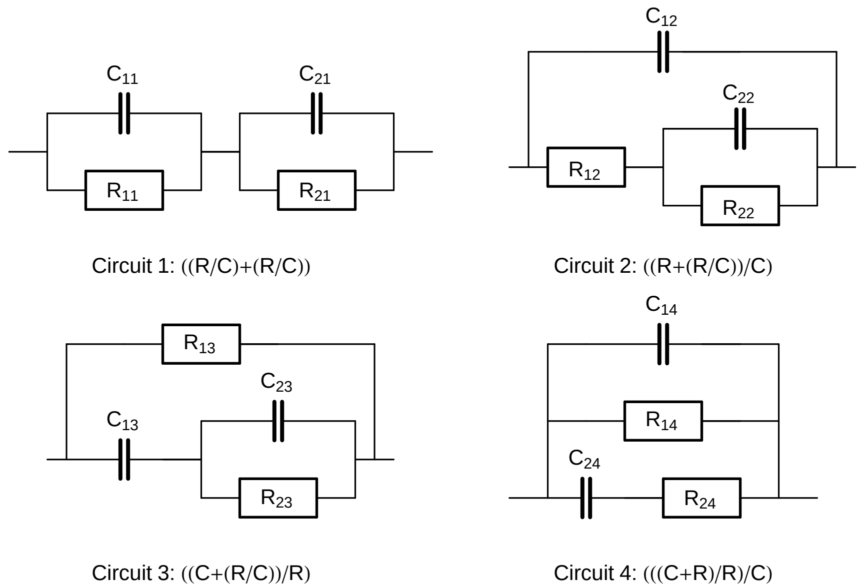

Figure 4.17: The four circuits are non-distinguishable [3, 2, 1, 6].

The four circuits are non-distinguishable [3, 2, 1, 6]. 12 transformation formulae exist between the four circuits.

4.5.1 Transformation formulae circuit 2 → circuit 1

$$C_{11} = \frac{1}{2\,C_{22}\,{R_{22}}^2}\sqrt{{C_{22}}^2\,{R_{22}}^2 + 2\,C_{12}\,C_{22}\,R_{22}\,\left(-R_{12}+R_{22}\right) + {C_{12}}^2\left(R_{12}+R_{22}\right)^2}$$
$$\times \left(C_{22}\,R_{22} - C_{12}\,\left(R_{12}+R_{22}\right) + \right.$$
$$\left.\sqrt{{C_{22}}^2\,{R_{22}}^2 + 2\,C_{12}\,C_{22}\,R_{22}\,\left(-R_{12}+R_{22}\right) + {C_{12}}^2\left(R_{12}+R_{22}\right)^2}\right)$$

$$C_{21} = \frac{1}{2\,C_{22}\,{R_{22}}^2}\sqrt{{C_{22}}^2\,{R_{22}}^2 + 2\,C_{12}\,C_{22}\,R_{22}\,\left(-R_{12}+R_{22}\right) + {C_{12}}^2\left(R_{12}+R_{22}\right)^2}$$
$$\times \left(-C_{22}\,R_{22} + C_{12}\,\left(R_{12}+R_{22}\right) + \right.$$
$$\left.\sqrt{{C_{22}}^2\,{R_{22}}^2 + 2\,C_{12}\,C_{22}\,R_{22}\,\left(-R_{12}+R_{22}\right) + {C_{12}}^2\left(R_{12}+R_{22}\right)^2}\right)$$

$$R_{21} = \left(C_{22}\,(R_{12} - R_{22})\,R_{22} - C_{12}\,(R_{12} + R_{22})^2 + (R_{12} + R_{22})\,\sqrt{{C_{22}}^2\,{R_{22}}^2 + 2\,C_{12}\,C_{22}\,R_{22}\,(-R_{12} + R_{22}) + {C_{12}}^2\,(R_{12} + R_{22})^2}\right) / \left(2\,\sqrt{{C_{22}}^2\,{R_{22}}^2 + 2\,C_{12}\,C_{22}\,R_{22}\,(-R_{12} + R_{22}) + {C_{12}}^2\,(R_{12} + R_{22})^2}\right)$$

$$R_{11} = \frac{1}{2}\left(R_{12} + R_{22} + \frac{C_{22}\,R_{22}\,(-R_{12} + R_{22}) + C_{12}\,(R_{12} + R_{22})^2}{\sqrt{{C_{22}}^2\,{R_{22}}^2 + 2\,C_{12}\,C_{22}\,R_{22}\,(-R_{12} + R_{22}) + {C_{12}}^2\,(R_{12} + R_{22})^2}}\right)$$

4.5.2 Transformation formulae circuit $1 \to$ circuit 2

$$C_{12} = \frac{C_{11}\,C_{21}}{C_{11} + C_{21}}, C_{22} = \frac{\left({C_{11}}^2\,R_{11} + {C_{21}}^2\,R_{21}\right)^2}{(C_{11} + C_{21})\,(C_{11}\,R_{11} - C_{21}\,R_{21})^2}$$

$$R_{12} = \frac{(C_{11} + C_{21})^2\,R_{11}\,R_{21}}{{C_{11}}^2\,R_{11} + {C_{21}}^2\,R_{21}}, R_{22} = \frac{(C_{11}\,R_{11} - C_{21}\,R_{21})^2}{{C_{11}}^2\,R_{11} + {C_{21}}^2\,R_{21}}$$

4.5.3 Transformation formulae circuit $3 \to$ circuit 1

$$C_{11} = \frac{1}{2\,{C_{13}}^2\,{R_{13}}^2}\left({C_{13}}^2\,(C_{13} + C_{23})\,{R_{13}}^2 + 2\,C_{13}\,\left({C_{13}}^2 - {C_{23}}^2\right)\,R_{13}\,R_{23} + (C_{13} + C_{23})^3\,{R_{23}}^2 - \left(C_{13}\,(C_{13} - C_{23})\,R_{13} + (C_{13} + C_{23})^2\,R_{23}\right) \times\sqrt{{C_{23}}^2\,{R_{23}}^2 + 2\,C_{13}\,C_{23}\,R_{23}\,(-R_{13} + R_{23}) + {C_{13}}^2\,(R_{13} + R_{23})^2}\right)$$

$$C_{21} = \frac{1}{2\,{C_{13}}^2\,{R_{13}}^2}\left({C_{13}}^2\,(C_{13} + C_{23})\,{R_{13}}^2 + 2\,C_{13}\,\left({C_{13}}^2 - {C_{23}}^2\right)\,R_{13}\,R_{23} + (C_{13} + C_{23})^3\,{R_{23}}^2 + \left(C_{13}\,(C_{13} - C_{23})\,R_{13} + (C_{13} + C_{23})^2\,R_{23}\right) \times\sqrt{{C_{23}}^2\,{R_{23}}^2 + 2\,C_{13}\,C_{23}\,R_{23}\,(-R_{13} + R_{23}) + {C_{13}}^2\,(R_{13} + R_{23})^2}\right)$$

$$R_{21} = R_{13}\,\left(C_{13}\,R_{13} - (C_{13} + C_{23})\,R_{23} + \sqrt{{C_{23}}^2\,{R_{23}}^2 + 2\,C_{13}\,C_{23}\,R_{23}\,(-R_{13} + R_{23}) + {C_{13}}^2\,(R_{13} + R_{23})^2}\right) / \left(2\,\sqrt{{C_{23}}^2\,{R_{23}}^2 + 2\,C_{13}\,C_{23}\,R_{23}\,(-R_{13} + R_{23}) + {C_{13}}^2\,(R_{13} + R_{23})^2}\right)$$

$$R_{11} = \frac{R_{13}}{2}\left(1 + \frac{-C_{13}\,R_{13} + (C_{13} + C_{23})\,R_{23}}{\sqrt{{C_{23}}^2\,{R_{23}}^2 + 2\,C_{13}\,C_{23}\,R_{23}\,(-R_{13} + R_{23}) + {C_{13}}^2\,(R_{13} + R_{23})^2}}\right)$$

4.5.4 Transformation formulae circuit $1 \rightarrow$ circuit 3

$$R_{13} = R_{11} + R_{21}, C_{23} = \frac{C_{11}\,C_{21}\,\left(C_{11}\,{R_{11}}^2 + C_{21}\,{R_{21}}^2\right)}{\left(C_{11}\,R_{11} - C_{21}\,R_{21}\right)^2}$$

$$C_{13} = \frac{C_{11}\,{R_{11}}^2 + C_{21}\,{R_{21}}^2}{\left(R_{11} + R_{21}\right)^2}, R_{23} = \frac{R_{11}\,R_{21}\,\left(R_{11} + R_{21}\right)\,\left(C_{11}\,R_{11} - C_{21}\,R_{21}\right)^2}{\left(C_{11}\,{R_{11}}^2 + C_{21}\,{R_{21}}^2\right)^2}$$

4.5.5 Transformation formulae circuit $4 \rightarrow$ circuit 1

$$C_{11} = \frac{1}{2\,C_{24}\,{R_{14}}^2}\sqrt{{C_{14}}^2\,{R_{14}}^2 + 2\,C_{14}\,C_{24}\,R_{14}\,\left(R_{14} - R_{24}\right) + {C_{24}}^2\left(R_{14} + R_{24}\right)^2} \times \left(C_{14}\,R_{14} - C_{24}\,\left(R_{14} + R_{24}\right) + \sqrt{{C_{14}}^2\,{R_{14}}^2 + 2\,C_{14}\,C_{24}\,R_{14}\,\left(R_{14} - R_{24}\right) + {C_{24}}^2\left(R_{14} + R_{24}\right)^2}\right)$$

$$C_{21} = \frac{1}{2\,C_{24}\,{R_{14}}^2}\sqrt{{C_{14}}^2\,{R_{14}}^2 + 2\,C_{14}\,C_{24}\,R_{14}\,\left(R_{14} - R_{24}\right) + {C_{24}}^2\left(R_{14} + R_{24}\right)^2} \times \left(-C_{14}\,R_{14} + C_{24}\,\left(R_{14} + R_{24}\right) + \sqrt{{C_{14}}^2\,{R_{14}}^2 + 2\,C_{14}\,C_{24}\,R_{14}\,\left(R_{14} - R_{24}\right) + {C_{24}}^2\left(R_{14} + R_{24}\right)^2}\right)$$

$$R_{21} = \left(R_{14}\,\left(\left(C_{14} + C_{24}\right)\,R_{14} - C_{24}\,R_{24} + \sqrt{{C_{14}}^2\,{R_{14}}^2 + 2\,C_{14}\,C_{24}\,R_{14}\,\left(R_{14} - R_{24}\right) + {C_{24}}^2\left(R_{14} + R_{24}\right)^2}\right)\right) \Big/ \left(2\sqrt{{C_{14}}^2\,{R_{14}}^2 + 2\,C_{14}\,C_{24}\,R_{14}\,\left(R_{14} - R_{24}\right) + {C_{24}}^2\left(R_{14} + R_{24}\right)^2}\right)$$

$$R_{11} = \frac{R_{14}}{2}\left(1 + \frac{-\left(C_{14} + C_{24}\right)\,R_{14} + C_{24}\,R_{24}}{\sqrt{{C_{14}}^2\,{R_{14}}^2 + 2\,C_{14}\,C_{24}\,R_{14}\,\left(R_{14} - R_{24}\right) + {C_{24}}^2\left(R_{14} + R_{24}\right)^2}}\right)$$

4.5.6 Transformation formulae circuit $1 \rightarrow$ circuit 4

$$C_{14} = \frac{C_{11}\,C_{21}}{C_{11} + C_{21}}, R_{14} = R_{11} + R_{21}$$

$$R_{24} = \frac{\left(C_{11} + C_{21}\right)^2 R_{11}\,R_{21}\,\left(R_{11} + R_{21}\right)}{\left(C_{11}\,R_{11} - C_{21}\,R_{21}\right)^2}, C_{24} = \frac{\left(C_{11}\,R_{11} - C_{21}\,R_{21}\right)^2}{\left(C_{11} + C_{21}\right)\,\left(R_{11} + R_{21}\right)^2}$$

4.5.7 Transformation formulae circuit $3 \rightarrow$ circuit 2

$$C_{12} = \frac{C_{13}\,C_{23}}{C_{13} + C_{23}}, C_{22} = \frac{\left({C_{13}}^2\,R_{13} + \left(C_{13} + C_{23}\right)^2 R_{23}\right)^2}{{C_{13}}^2\,\left(C_{13} + C_{23}\right)\,{R_{13}}^2}$$

$$R_{12} = \frac{\left(C_{13} + C_{23}\right)^2 R_{13}\,R_{23}}{{C_{13}}^2\,R_{13} + \left(C_{13} + C_{23}\right)^2 R_{23}}, R_{22} = \frac{{C_{13}}^2\,{R_{13}}^2}{{C_{13}}^2\,R_{13} + \left(C_{13} + C_{23}\right)^2 R_{23}}$$

4.5.8 Transformation formulae circuit $2 \to$ circuit 3

$$R_{13} = R_{12} + R_{22}, C_{23} = \frac{C_{12}\,\left(C_{22}\,{R_{22}}^2 + C_{12}\,(R_{12} + R_{22})^2\right)}{C_{22}\,{R_{22}}^2}$$

$$C_{13} = C_{12} + \frac{C_{22}\,{R_{22}}^2}{(R_{12} + R_{22})^2}, R_{23} = \frac{{C_{22}}^2\,R_{12}\,{R_{22}}^3\,(R_{12} + R_{22})}{\left(C_{22}\,{R_{22}}^2 + C_{12}\,(R_{12} + R_{22})^2\right)^2}$$

4.5.9 Transformation formulae circuit $4 \to$ circuit 2

$$C_{12} = C_{14}, C_{22} = \frac{C_{24}\,(R_{14} + R_{24})^2}{{R_{14}}^2}, R_{12} = \frac{R_{14}\,R_{24}}{R_{14} + R_{24}}, R_{22} = \frac{{R_{14}}^2}{R_{14} + R_{24}}$$

4.5.10 Transformation formulae circuit $2 \to$ circuit 4

$$C_{14} = C_{12}, R_{14} = R_{12} + R_{22}, R_{24} = R_{12} + \frac{{R_{12}}^2}{R_{22}}, C_{24} = \frac{C_{22}\,{R_{22}}^2}{(R_{12} + R_{22})^2}$$

4.5.11 Transformation formulae circuit $4 \to$ circuit 3

$$R_{13} = R_{14}, C_{23} = C_{14} + \frac{{C_{14}}^2}{C_{24}}, C_{13} = C_{14} + C_{24}, R_{23} = \frac{{C_{24}}^2\,R_{24}}{(C_{14} + C_{24})^2}$$

4.5.12 Transformation formulae circuit $3 \to$ circuit 4

$$C_{14} = \frac{C_{13}\,C_{23}}{C_{13} + C_{23}}, R_{24} = \frac{(C_{13} + C_{23})^2\,R_{23}}{{C_{13}}^2}, R_{14} = R_{13}, C_{24} = \frac{{C_{13}}^2}{C_{13} + C_{23}}$$

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