How to fit transmission lines with ZFit

I – INTRODUCTION

ZFit is the EC-Lab®. impedance fitting tool. This note will describe how to fit transmission lines using one equivalent circuit elements contained in ZFit.

It has long been known that the Warburg impedance is equivalent to that of a semi-infinite large network i.e. a transmission line, as shown in Fig. 1 [1, 2].

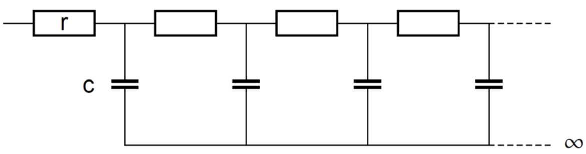

Figure 1: The equivalent circuit of the Warburg impedance.

More recently it has been shown [3] that the impedance of a L-long transmission line made of χ and ζ elements and terminated by a Z_L element (Fig. 2) is given by the general expression:

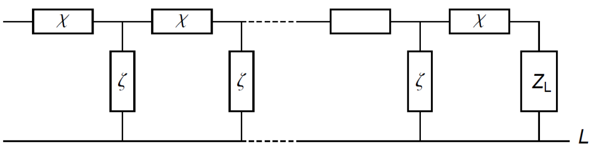

Figure 2: Uniform transmission line made χ and ζ elements and terminated by Z_L [3].

$$Z = \frac{(\zeta\chi - Z_L^2)\,\mathrm{sh}\left(\frac{L\sqrt{\chi}}{\sqrt{\zeta}}\right)}{Z_L\,\mathrm{sh}\left(\frac{L\sqrt{\chi}}{\sqrt{\zeta}}\right) + \sqrt{\zeta\chi}\,\mathrm{ch}\left(\frac{L\sqrt{\chi}}{\sqrt{\zeta}}\right)} + Z_L$$

With three limiting cases

- open-circuited transmission line

$$Z_L = \infty \Rightarrow Z = \sqrt{\zeta\chi}\,\coth\left(\frac{L\sqrt{\chi}}{\sqrt{\zeta}}\right) \quad (1)$$

- short-circuited transmission line

$$Z_L = 0 \Rightarrow Z = \sqrt{\zeta\chi}\,\mathrm{th}\left(\frac{L\sqrt{\chi}}{\sqrt{\zeta}}\right) \quad (2)$$

- semi-infinite transmission line

$$L \to \infty \Rightarrow Z = \sqrt{\zeta\chi} \quad (3)$$

Hereafter, some transmission lines are described and the corresponding ”simple” equivalent circuit elements are shown. Firstly, the open-circuited transmission lines will be explained, followed by short-circuited and semi-infinite transmission lines[1].

III – OPEN-CIRCUITED TRANSMISSION LINES Z_L = 1

II - 1 OPEN-CIRCUITED URC (UNIFORM DISTRIBUTED RC)

Let us consider the open-circuited transmission line made of r and c elements (Fig. 3).

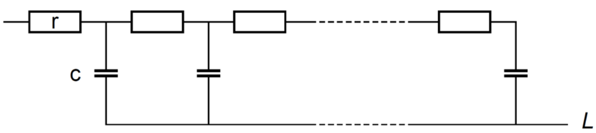

Figure 3: L-long open uniform distributed RC (URC) transmission line [4,5].

[1] The transmission lines are named accordingly to the U-χζ format where U means uniformly distributed and χ and ζ are the elements of the transmission line.

Using Eq. (1), the transmission line impedance is given by:

$$\chi = r, \zeta = \frac{1}{j\omega c} \Rightarrow Z = \sqrt{r}\frac{\coth\left(L\sqrt{rcj\omega}\right)}{\sqrt{cj\omega}}$$

(4)

With $\omega = 2\pi f$. This impedance is similar to that of the M element of ZFit

$$Z_M = R_d \frac{\coth\sqrt{\tau_d j\omega}}{\sqrt{\tau_d j\omega}}, R_d = L_r, \tau_d = L^2 rc$$

(5)

II - 2 OPEN-CIRCUITED URQ

Replacing c elements by q elements, with $Z_q = 1 \Big/ \left(q\left(j\omega\right)^{\alpha}\right)$ leads to transmission line shown in Fig. 4.

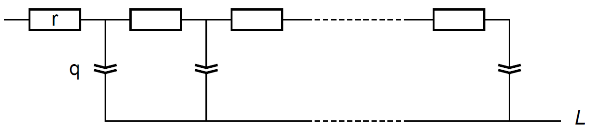

Figure 4: L-long open uniform distributed RQ|(URQ) transmission line.

The transmission line impedance is given by

$$\chi = r, \zeta = \frac{1}{q\left(j\omega\right)^{\alpha}}$$
$$\Rightarrow Z = \sqrt{r}\frac{\coth\left(L\sqrt{rq}\left(j\omega\right)^{\alpha/2}\right)}{\sqrt{q}\left(j\omega\right)^{\alpha/2}} \quad (6)$$

The impedance is similar to that of the M_a element of ZFit.

$$Z_{M_a} = R\frac{\coth\left(\tau j\omega\right)^{\alpha/2}}{\left(\tau j\omega\right)^{\alpha/2}} \quad (7)$$

With $R = Lr, \tau = \left(L^2 rq\right)^{1/\alpha}$

For example, a Nyquist impedance diagram of a battery Ni-MH 1900 mAh is shown in Fig. 5. The equivalent circuit R1+L1+Q1/(R2+Ma3), containing a M_a element, is chosen to fit the data shown in Fig. 5. The values of the parameters, obtained using the EC-Lab ZFit tool , are $R1$ = 0.049 Ω, $L1$ = 0.154 10^{-6} H, $Q1$ = 0.66 F $s^{\alpha-1}$, $\alpha 1$ = 0.61, $R2$ = 0.0236 Ω, $R3$ = L r = 0.057 Ω, $\tau 3 = (L^2\ r\ q)^{1/\alpha}$ = 2.25 s and $\alpha 3$ = 0.89.

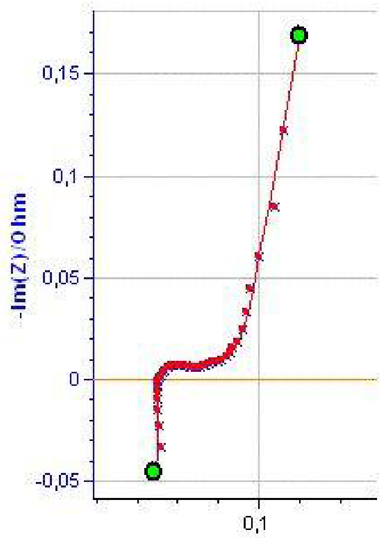

Figure 5: Nyquist impedance diagram of a battery Ni-MH 1900 mAh.

The equivalent circuit of the anomalous diffusion is shown in Fig. 6 [6].

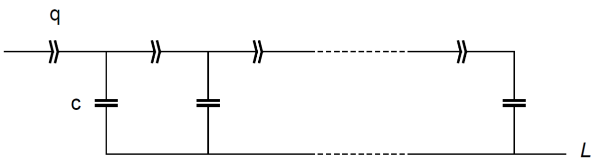

Figure 6: L-long open uniform distributed QC (UQC) transmission line. Anomalous diffusion [6].

The anomalous diffusion impedance is given by

$$\chi = \frac{1}{q\left(j\omega\right)^{\alpha}}, \zeta = \frac{1}{cj\omega}$$

$$\Rightarrow Z = \frac{\coth\left(L\sqrt{\frac{c}{q}}\left(j\omega\right)^{\frac{1}{2}-\frac{\alpha}{2}}\right)}{\sqrt{cq}\left(j\omega\right)^{\frac{\alpha}{2}+\frac{1}{2}}} \quad (8)$$

This impedance is similar to that of the M_g element of ZFit

$$Z_{M_g} = R\frac{\coth\left(\tau j\omega\right)^{\gamma/2}}{\left(\tau j\omega\right)^{1-\gamma/2}} \quad (9)$$

With $\gamma = 1-\alpha$, $R = c^{\frac{1}{\gamma}-1} L^{\frac{2}{\gamma}-1} q^{\frac{-1}{\gamma}}$, $\tau = c^{\frac{1}{\gamma}} L^{\frac{2}{\gamma}} q^{\frac{-1}{\gamma}}$.

III – SHORT-CIRCUITED TRANSMISSION LINES Z_L=0

III - 1 SHORT-CIRCUITED URC

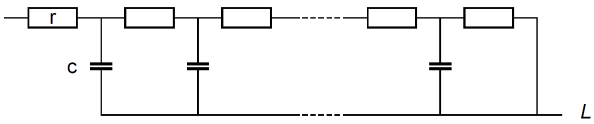

Figure 7: L-long open uniform distributed RC (URC) transmission line.

Using Eq. (2), the impedance of the short-circuited transmission line made of r and c elements (Fig. 7) is given by

$$\chi = r, \zeta = \frac{1}{cj\omega}$$

$$\Rightarrow Z = r\frac{\mathrm{th}\left(L\sqrt{rcj\omega}\right)}{\sqrt{rcj\omega}} \quad (10)$$

This impedance is similar to that of the W_d element of ZFit

$$Z_{W_d} = R_d\frac{\mathrm{th}\sqrt{\tau_d j\omega}}{\sqrt{\tau_d j\omega}}, R_d = Lr, \tau_d = L^2 rc \quad (11)$$

IV – SEMI-INFINITE TRANSMISSION LINES: $L \to \infty$

IV - 1 SEMI-INFINITE URC

The impedance of the semi-infinite transmission line shown in Fig. 1 is obtained making $L \to \infty$ in Eq. (10).

$$L \to \infty \Rightarrow Z = r\frac{\mathrm{th}\left(L\sqrt{rcj\omega}\right)}{\sqrt{\sqrt{rcj\omega}}} \approx \frac{\sqrt{r}}{\sqrt{cj\omega}} \quad (12)$$

This expression is similar to that of the Warburg (W) element of ZFit

$$Z_W = \frac{2\sigma}{\sqrt{j\omega}} \text{ with } \sigma = \frac{\sqrt{r}}{2\sqrt{c}} \quad (13)$$

As an example a Nyquist impedance diagram of a Fe(II)/Fe(III) system is shown in Fig. 8.

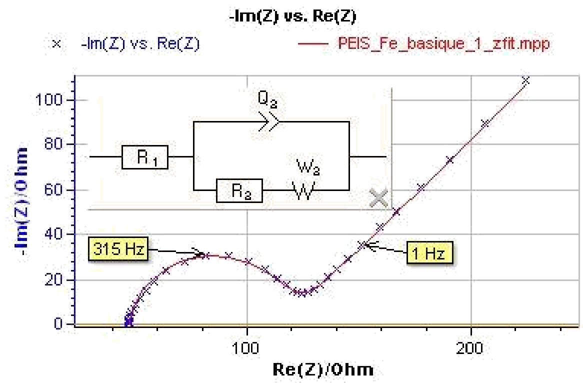

Figure 8: Nyquist impedance diagram of a Fe(III)/Fe(II) system in basic medium.

The Randles circuit R1+Q2/(R2+W2), containing a Warburg element, is chosen to fit the data shown in Fig. 8. The values of the parameters for equivalent circuit are R_1 = 47.57 Ω, Q_2 = 17.09 x 10^{-6} F s^{-1}, α = 0.885, R_2 = 70.94 Ω and σ_2 = 85.33 Ω $s^{-1/2}$

$$\Rightarrow \sqrt{\frac{r}{c}} = 42.7 \ \Omega \ s^{-1/2}.$$

IV - 2 SEMI-INFINITE URRC

First of all, let us calculate the impedance of the L-long URRC transmission line (Fig. 9) corresponding to diffusion-reaction and diffusion-trapping impedance [7]:

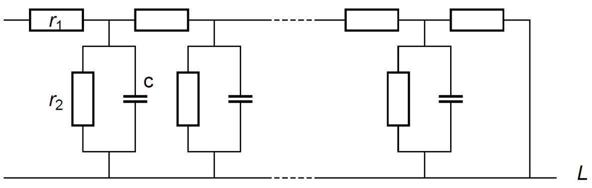

Figure 9: L-long short-circuited uniform distributed RRC (URRC) transmission line.

$$\chi = r_1, \zeta = \frac{r_2}{1 + r_2 c j\omega}$$

$$\Rightarrow Z = \sqrt{r_1 r_2}\,\frac{\mathrm{th}\left(L\sqrt{\frac{r_1}{r_2}\left(1 + r_2 c j\omega\right)}\right)}{\sqrt{1 + r_2 c j\omega}} \qquad (14)$$

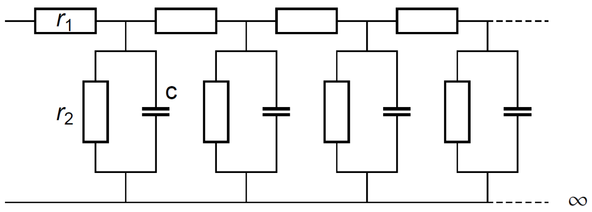

Figure 10: Semi-infinite short-circuited uniform distributed RRC (URRC) transmission line.

With $L \to \infty$ it is obtained [8]:

$$L \to \infty \Rightarrow Z \approx \frac{\sqrt{r_1 r_2}}{\sqrt{1 + r_2 c j\omega}} \qquad (15)$$

This expression is similar to that of the Gerischer element G of ZFit [9]:

$$Z_G = \frac{R_G}{\sqrt{1 + \tau_G j\omega}}, R_G = \sqrt{r_1 r_2}, \tau_G = r_2 c \qquad (16)$$

IV - 3 SEMI-INFINITE URRQ

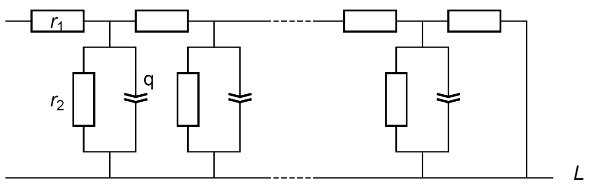

Figure 11: L-long short-circuited uniform distributed RRQ (URRQ) transmission line.

Replacing c elements by q elements

$$\chi = r_1, \zeta = \frac{r_2}{1 + r_2 c\left(j\omega\right)^{\alpha}}$$

$$\Rightarrow Z = \sqrt{r_1 r_2}\,\frac{\mathrm{th}\left(L\sqrt{\frac{r_1}{r_2}\left(1 + r_2 c\left(j\omega\right)^{\alpha}\right)}\right)}{\sqrt{1 + r_2 c\left(j\omega\right)^{\alpha}}} \qquad (17)$$

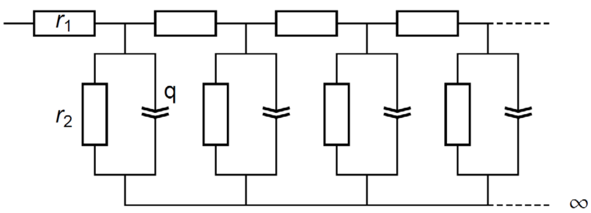

Figure 12: Semi-infinite short-circuited uniform distributed RRQ (URRQ) transmission line.

And $L \to \infty \Rightarrow Z \approx \frac{\sqrt{r_1 r_2}}{\sqrt{1 + \tau\left(j\omega\right)^{\alpha}}}$ (18)

This expression is similar to that of the G_a element of ZFit

$$Z_{G_a} = \frac{R}{\sqrt{1 + \tau\left(j\omega\right)^{\alpha}}}, R = \sqrt{r_1 r_2}, \tau = r_2 q \qquad (19)$$

V – CONCLUSION

Seven elements, W, Wd, M, Ma, Mg, G and Ga, available in ZFit correspond to different transmission lines (Tabs. I and II).

Table I: Summary table.

Transmission line		ZFit Element
Open Circuited	URC	M
	URQ	Ma
	UQC	Mg
Short circuited	URC	Wd
Semi-infinite	URC	W
	URRC	G
	URRQ	Ga

Data files can be found in : C:\Users\xxx\Documents\EC-Lab\Data\Samples\EIS\PEIS_Fe_Basique_1 *and AN43_peis_batteries_carouf_01_PEIS_C06*

REFERENCES

1) J. C. Wang, J. Electrochem. Soc., 134 (1987) 1915.
2) M. Sluyters-Rehbach, Pure & Appl. Chem., 66 (1994) 1831.
3) J. Bisquert, Phys. Chem. Chem. Phys., 2 (2000) 4185.
4) G. C. Temes and J. W. Lapatra, *Introduction to Circuits Synthesis and Design*, McGraw-Hill, New-York, 1977.
5) J.-P. Diard, B. Le Gorrec, and C. Montella, J. Electroanal. Chem., 471 (1999) 126.
6) J. Bisquert and A. Compte, J. Electroanal. Chem., 499 (2001) 112.
7) J.-P. Diard and C. Montella, J. Electroanal. Chem., 557 (2003) 19.
8) B. A. Boukamp and H. J.-M. Bouwmeester, Solid State Ionics 157 (2003) 29.
9) H. Gerischer, Z. Physik. Chem. (Leipzig), 198 (1951) 286.

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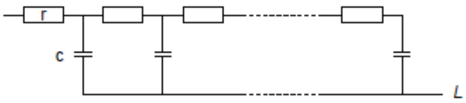
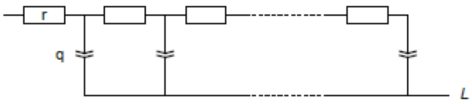
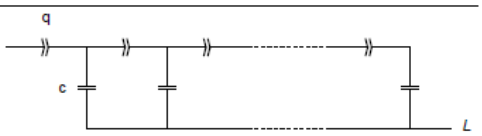
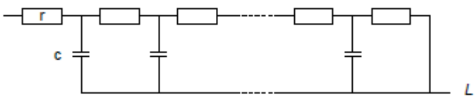
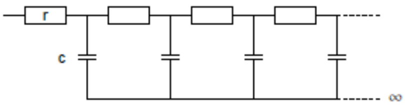
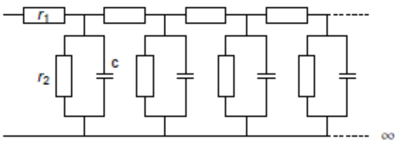
Table II: ZFit elements *vs.* transmission lines

ZFit element	Equations	Transmission line
M	$R_d \dfrac{\coth\sqrt{\tau_d j\omega}}{\sqrt{\tau_d j\omega}}$ $R_d = L\,r,\ \ \tau_d = L^2\,r\,c$	
M_a	$R\dfrac{\coth(\tau j\omega)^{\alpha/2}}{(\tau j\omega)^{\alpha/2}}$ $R = L\,r$ $\tau = (L^2\,r\,q)^{1/\alpha}$	
M_g	$R\dfrac{\coth(\tau j\omega)^{\gamma/2}}{(\tau j\omega)^{1-\gamma/2}}$ $R = c^{\frac{1}{\gamma}-1}L^{\frac{2}{\gamma}-1}q^{-1/\gamma}$ $\tau = c^{\frac{1}{\gamma}}L^{2/\gamma}q^{-1/\gamma}$	
W_d	$R_d \dfrac{\text{th}\sqrt{\tau_d j\omega}}{\sqrt{\tau_d j\omega}}$ $R_d = L\,r$ $\tau_d = L^2\,r\,c$	
W	$\dfrac{2\sigma}{\sqrt{j\omega}}$ $\sigma = \dfrac{\sqrt{r}}{2\sqrt{c}}$	
G	$\dfrac{R_G}{\sqrt{1+\tau_G j\omega}}$ $R_G = \sqrt{r_1 r_2}$ $\tau_G = r_2\,c$	
G_a	$\dfrac{R_G}{\sqrt{1+\tau_G (j\omega)^{\alpha}}}$ $R_G = \sqrt{r_1 r_2}$ $\tau_G = r_2\,q$